

RELIABILITY BASED DESIGN OF R C C BEAMS WITH DURABILITY CONSIDERATIONS

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in Partial Fulfilment of the Requirements
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DOCTOR OF PHILOSOPHY

By
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to the

DEPARTMENT OF CIVIL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR
DECEMBER, 1982

DEDICATED

TO

MY YOUNGER BROTHER 'VENU'

WHOM GOD LOVED TOO EARLY

CERTIFICATE

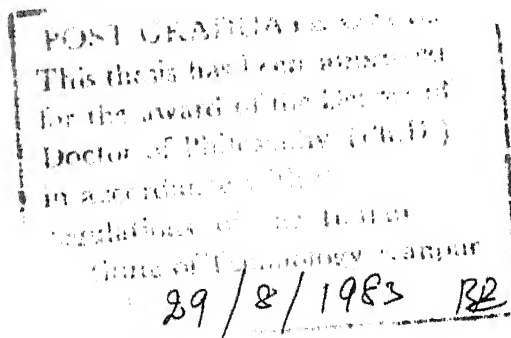
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NOTATIONS

A	Regression constant
A_{acid}	Amount of acid
AN	Acid number
A/C	Aggregate cement ratio
A_{st}	Area of tension steel
B	Regression coefficient
b	Breadth of beam
C	Regression constant
C_u	Total compressive force
C_x	Coefficient of variation of a random variable x
c	Strength of acid
c_t	Moles of acid/cc at time t
d	Effective depth
D	Overall depth
E	Event
E_s	Modulus of elasticity of steel
$F_x(x)$	Probability distribution function of a random variable x
f_{ck}	Characteristic strength of a concrete
f_{co}	Strength of exposed concrete
$f(i)$	Probability of failure in i th year for the first time
f_y	Characteristic strength of steel
$f_x(x)$	Probability density function of a random variable x
$H (\quad)$	Hazard rate

K	$0.36 f_{ck} b d^2$
k_b	Ratio of critical neutral axis depth to effective depth
M_e, m_e	External moment
M_r, m_r	Moment of resistance
M_u	Ultimate moment capacity without partial safety factors
M_x	Molecular weight of an element x
N	Normality
P_f	Probability of failure
P_x	Percentage of a compound x
$P[]$	Probability of []
p_i	Probability of failure in the ith instance
p_t	Percentage tension reinforcement
Q_x	Quantity of an element x
q	Quantity of cement in 1 m^3 of concrete
R	Reliability
r_f	Strength reduction factor
r_t	Rate of reaction
r_w	Weight reduction factor
s_x	Standard deviation of x
T	Total tensile force
t	Time in years
t_d	Depth of deterioration of concrete
t_p	Depth of penetration of acid

V	Volume of acid consumed per unit surface area
V_b	Volume of base
V_R	Amount of acid required for saturated reaction with one unit of cement
V_s	Volume of sample
W/C	Water cement ratio
w_{ce}	Weight reduction factor
w_{co}	28 days weight of concrete
x_c	Critical neutral axis depth
x_u	Neutral axis depth of an unexposed beam
x_{ue}	Neutral axis depth of an exposed beam
Z	Standard normal variate
Φ	Standard normal probability function
η	Error
ϕ	Diameter of the bar
φ	Function of Random Variables.

SYNOPSIS

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Concrete is a durable building material and has wide applications. But when it is exposed to aggressive environment, deterioration in the material takes place which results into a reduced life of the structure. Acidic environment is created in fertilizer factories, tanneries, sewer pipes etc. It also occurs when some exhaust gasses combine with atmosphere. Acid reacts with cement compounds and degenerates the product. Literature on : durability of concrete exposed to sulphates, chlorides, acids, seawater; and probabilistic distributions of strength of concrete and beams; safety of structures, risk analysis and design of members is reviewed. Chemistry of cement, compound formation, hydration of cement, heat of

hydration of cement; and corrosion mechanisms of concrete and reinforcement are discussed at length for better understanding of the problem. The stoichiometrical relations of some of the chemicals with the hydrated cement compounds which facilitate to estimate the amount of a particular chemical required for a saturated reaction with cement are presented.

An experimental investigation was carried out to estimate the deterioration of concrete when exposed to sulphuric acid of different concentrations. Mathematical expressions are developed to estimate; the amount of acid consumed by concrete per unit surface area with time, strength and weight reduction in concrete, and depth of deterioration of concrete for a given amount of acid consumed. These expressions are useful in proportioning a member and to estimate its life, and provide proper cover to main reinforcement etc.

The influence of deteriorated concrete on moment capacity of beam is studied. The rate of decrease of moment capacity is low in under - reinforced sections when compared with that in over - reinforced sections. Strength of a member and loading on it are random in nature, so it is reasonable to consider the probabilistic variations in the design. All the variables are assumed as Gaussian random variables and a procedure for estimating the coefficient of variation of moment capacity is explained. A comparison of

design of reinforced concrete beams exposed and unexposed (exposed to normal environment) by both deterministic and probabilistic approaches is made. The influence of preassigned probability of failure on cost of beam is discussed. The cost increases with decrease in probability of failure, but the rate at which it increases is higher for higher probability of failures.

A procedure for estimating the probability of failure, reliability and hazard rate of a deteriorating member is presented along with illustrative examples. Four cases with illustrative examples are studied. The first one is where the probability of failure in any year is constant. In the second case, the probability of failure remains constant upto 20 years, but increases due to sudden reduction in strength or cracking of a beam. Case three is similar to case two except that the load increases instead of strength reduction. In the fourth case, a time dependent deterioration in strength of concrete is considered. Cumulative probability of failure at the end of 50 years is calculated for the four cases and comparison is made. The effect of survival for every 10 year period on the cumulative probability of failure is calculated. It is found that a 20% reduction in moment capacity increases the probability of failure from $3(10^{-10})$ to $42(10^{-6})$. Similarly a 20% increase in load-

ing increases the probability of failure from $3(10^{-10})$ to $7(10^{-6})$. A reduction of 45% strength of concrete increases the probability of failure from $3(10^{-13})$ to $8(10^{-9})$. The estimation of revised probability of failure incorporating the observed data is useful in predicting the total life of a structure.

CHAPTER I

INTRODUCTION

1.1 Introduction

Chemical environment is one of the factors which affects the durability of concrete. When exposed to some aggressive chemicals, the strength of concrete deteriorates, which ultimately leads to the premature failure of a member. The high alkalinity of concrete ($P^H \approx 13$) is tampered due to carbonation and leaching out of lime in some chemical reactions. The reduction in P^H allows the formation of rust on reinforcing steel. It is also known that corrosion of concrete results in two ways viz. a) swelling attack, and b) dissolving attack. Sulphates present in soil and water disintegrates the concrete due to swelling attack. Calcium aluminate compounds of hydrated cement form voluminous products when they react with sulphates. Their increase in volume weakens the internal frame work and splitting of concrete takes place. Acids dissolve the components of set cement. Particularly sulphuric acid is a serious threat to durability of concrete, because it has both dissolving attack as well as swelling attack. Sulphuric acid normally occurs in fertilizer factories, tannerics, sewer pipes etc.

Some exhaust gases when combined with atmosphere forms sulphuric acid.

Some authors [1, 2, 3, 4, 5]^{*} have quantified the deterioration of concrete under different chemical environments. Literature on the effect of sulphuric acid on concrete is limited. From the literature one can see that, the research is mostly oriented towards the effect of salts on concrete. But it is desirable to quantify the deterioration of concrete in acid environments also. The simple mathematical expressions developed in this thesis can be used for proper design of members exposed to sulphuric acid. Throughout the thesis, the term 'exposed' is used to denote the concrete exposed to sulphuric acid and 'unexposed' one to denote the concrete exposed to normal environment.

Most of the design variables in concrete structures are random in nature. Particularly, strength of concrete is a function of many uncertain parameters. Similarly loads are also random in nature. The reliability of a structure depends on the variability in resistance and external forces. The reliability of a member does not increase beyond a particular range of coefficients of variation, even if the ratio of moment capacity to external moment is increased to any higher value. Therefore a rationalized design should incorporate the statistical

* The numerals in brackets indicate the reference numbers given at the end.

variations associated with design variables and external loads also.

1.2 Literature Review

1.2.1 Introduction

The review of literature is divided into three groups. First group is of the effect of various aggressive chemicals on concrete. Second one is on the mathematical modelling of deterioration of concrete and the third is on probabilistic formulation of design of concrete members. The aggressive chemical action of calcium chloride, sodium chloride and magnesium chloride among chlorides, magnesium and sodium sulphates among sulphates, ~~seawater and~~ sulphuric acid among acids is reviewed. The literature on sulphuric acid is scarcely available. Literature on probabilistic distributions for strength of concrete and beams, safety of structures, risk analysis, probabilistic design of members and application of decision theory is reviewed.

1.2.2 Effect of Chlorides on Concrete

If calcium chloride reacts with concrete, the calcium ion common to both calcium chloride and calcium hydroxide produces a large increase in calcium ion concentration and correspondingly decreases the hydroxyl ion concentration [6]. Berman [6] suggested that sodium chloride of

less than 0.003 molar does not contribute to the solubility of calcium hydroxide, but addition of sodium chloride upto about 1 molar increases the solubility of calcium hydroxide and a minimum of 0.03 molar sodium chloride was found necessary to produce active corrosion in reinforced steel. He has also reported that there is a decrease in P^H value when sodium chloride is added to the saturated lime solution. In contradiction to the above, Gjorv and Vennesland [7] have stated that as much as 20 % concentration of sodium chloride did not reduce the P^H value and they reasoned that the decrease in P^H value as reported by Berman [6] could be due to an alkaline error of the P^H electrode used. They also suggest that the level of P^H should never drop below about 11.5 to provide good corrosion protection to embedded steel. Higher the P^H value, higher the amounts of chlorides that can be tolerated. In 1980, Powers and Hammersey [8] have reported a reduction in P^H value from 13 to 11 when 0.2% sodium chloride is added to aqueous cement extracts. Neville [9] states that the effect of sodium chloride on concrete is same as that of calcium chloride. But the former is of lower intensity and causes depression in heat of hydration, which consequently lowers the strength of concrete of age 7 days and older.

An experimental investigation was carried out by Smolezyk [10 , 11] using 40 mm cubes of concrete, water cured for 10 days and immersed for two years in strong solutions of sodium chloride, calcium chloride and magnesium chloride, to study the engineering and chemical properties of concrete. He found that the destroying type of reactions occurred in the later two, if concrete contained excess of free lime, but all the three chemically react with concrete. Concrete made with ordinary portland cement containing C_3A about 12 % , when immersed in saturated sodium chloride solution for two years reduced the compressive strength to 60% and 56% for water cement ratios 0.5 and 0.7 respectively as compared to that of water cured concrete. If the C_3A is 1% the corresponding strengths are 66 % and 56 % respectively. According to Biczok [12], both sodium chloride and calcium chloride increase the solubility of tricalcium sulphoaluminate hydrate. Gjorv and Vennesland [13] have concluded that the effect of water cement ratio on chloride content in a specimen is limited mainly to surface layers and that too, for short durations of exposure. For longer duration of exposure to chloride bearing solutions and penetrations at greater depths, water cement ratio has no influence. Chloride ion concentrations within the inner layers of concrete depend

upon the type of cement and ion exchange capacity of the matrix system. Midgley and Illston [14] investigated with water cured concrete cylinders (230 mm diameter and 700 mm high) immersed in water, sodium chloride of 30,000 and 1,50,000 ppm concentrations for a period of 6 months. Higher the water cement ratio, greater the penetration of chloride ions, was observed. The concentration with depth followed a power relationship. Smolezyk [10] has studied the effect of 3 mol/litre magnesium chloride solution on a concrete made with ordinary portland cement containing C_3A 12% and 1% . The cubes were immersed for two years after 10 days water curing , 3 days water curing and 7 days moist curing. The cubes showed deterioration within 4 months period, and the extent of deterioration was so much , no compressive strength test could be conducted for the concretes made with high C_3A . A concrete cube made with cement having 1% C_3A and 0.7 water cement ratio showed some strength but very low, that is less than 3% strength of water cured samples of identical nature. Dilute magnesium chloride (e.g. 1%) has no appreciable effect on ordinary portland cement [15] . Suzukawa [16] studied on 40x40x160 mm prisms for compressive strength and 25x25x28.6 mm prisms for expansion tests, made of cement sand mortar in 1:2 proportion and water cement ratio 0.6. Former were made

using ordinary portland cement and moderate heat cement. The specimens were immersed in magnesium chloride solution after 6 days of water curing in 1, 4 and 25 gms/litre of concentrations for 1, 4, 13, 26 and 52 weeks. Compressive strengths of 84, 64 and 47% for mortar made with ordinary portland cement at the immersion of 4, 26 and 52 weeks respectively, when immersed in 25 gms/litre concentration were observed. Residual strengths of 87, 63 and 58 % were observed in case of moderate heat cement in identical conditions.

1.2.3 Effect of Sulphates

Magnesium sulphate on reaction with set cement produces magnesium hydroxide which is soluble in water to the extent of 0.01 gm/litre and P^H value of saturated solution of magnesium hydroxide [15] is about 10.5. McMillar [17] observed the influence of magnesium sulphate and sodium sulphate solutions for 5 years on 150 mm cube , compressive strength behaviour of concrete made with mixes having cement content of 390, 307 and 223 kg/m³, and water cement ratio 0.4, 0.5 and 0.7 respectively for each of ASTM type I to V.

150 mm cubes of concrete made with ordinary portland cement with different mix proportions and water cured for

13 days were exposed to 5% magnesium sulphate [18] . A loss of strength 34 % in 1:1:2 concrete with water cement ratio 0.45 and 66.67 % in 1:2:4 concrete with water cement ratio 0.50 was observed at the end of 30 months immersion. Whereas a loss of strength of 65 % was observed in 1:3:6 concrete with water cement ratio 0.7 , when immersed for 20 months. Tensile strength of 1:3 mortar briquettes exposed to 0.75 molar solution of sodium sulphate increased in the first 7 or 8 days of exposure and then started decreasing to as much as 0.2 Mpa from 1.1 Mpa in 35 days of exposure [19] . According to Keene [20] no definite conclusion could be drawn as to the effect of aggregate cement ratio and resistance to sulphate attack, but it was observed that lower mixes appeared to have no less resistance than richer ones. On the contrary, the Cement Research Institute of India's [21] report says that two concrete mixes with water cement ratio 0.45 for cement content 330 kg/m^3 and water cement ratio 0.7 for cement content 260 kg/m^3 , when water cured for 45 days and exposed to 3.5% SO_3 concentration for a period of one year, suffered a loss of compressive strength of 3% in the former mix and 24% in the later one.

1.2.4 Effect of Seawater on Concrete

Concrete cylinders, 75 mm diameter and 150 mm high made with concretes having cement content 593, 356 and 213 kg/m^3

were exposed in the tidal zone of seawater at Sheerness and another identical set partially immersed in three times the concentration of normal seawater [15] . Tidal actions were also simulated in the second set of experiments.

Portland cement showed a variable behaviour which was not related to their calculated content of C_3A . Concrete made with ordinary portland cement (C_3A content 8 %) , with aggregate cement ratios 2.6 and 5 and corresponding water cement ratios of 0.35 and 0.5 showed a gain in compressive strengths of order 25 % and 7% respectively, when exposed to 3 times normal seawater salt concentration for 10 years. But concrete with A/C ratio 9.0 and W/C ratio 0.96 showed decrease in compressive strength of the order of 50 % .

Concrete with ordinary portland cement having 10% C_3A and having identical mix parameters as above suffered a loss of strength ranging from 80% to 90 % in 10 years. However pozzolanic portland blast-furnace and high-alumina cements, with A/C ratio 2.6 and 5 showed higher resistance to direct attack. A 67 years old plain concrete blocks partially submerged in seawater at Los Angeles, made with six different cements and three different concrete mixes with each cement were investigated by Mehta and Haynes [22]. Dense concretes made with even 14-15 % C_3A were in excellent condition, but lean concrete showed loss of material and was soft and weak. Concrete cubes made with mixes

1) W/C ratio 0.45 and cement content 370 kg/m^3 and 2) W/C 0.7 and cement content 270 kg/m^3 were immersed in artificial seawater for one year [21] . The cubes were water cured for 48 days before immersion in artificial seawater. A decrease in strength of 2 % in mix (1) and 15 % in mix (2) were observed.

1.2.5 Effect of Sulphuric Acid on Concrete

Glenn William DePuy [1] investigated the effect of sulphuric acid on unimpregnated (UIC) and polymer impregnated concretes (PIC) exposed to 5 % and 15% concentration of sulphuric acid and 15% concentration of hydrochloric acid. A concrete made with a cement content 307 kg/m^3 of Type II portland cement, with W/C ratio 0.51 and 6 % entrained air was used in the investigation. A loss of weight of 25 % was observed in 105 days when exposed to 15% hydrochloric acid, in 49 and 210 days when exposed to 15% and 5% concentrations of sulphuric acid respectively. A deterioration rate of 1.74 and 1.6 respectively on weight and compressive strength when exposed to 5% concentration of sulphuric acid was reported. Fukuchi and Ohama [23] have investigated the resistance of autoclaved (AC) and polymer impregnated autoclaved (PIAC) concrete exposed to sulphuric acid. It was found that the chemical resistance is in following order from good to bad: PIAC, AC and water

cured concrete. The superior chemical resistance of PIAC is due to the impregnation of autoclaved concrete with chemically resistant polymer. Hughes and Guest [24] have investigated the relative rate of damage of limestone aggregate and siliceous gravelstone aggregate concretes exposed to sulphuric acid of concentrations 0.0016% to 0.02% . The concrete surface remained smooth throughout the erosion process in the former concrete and irregular surface was formed in the later one. The acid consumption was 4 to 5 times greater for the limestone concrete.

1.2.6 Mathematical Modelling of Deterioration of Concrete

Glenn William DePuy [1] has developed durability indices: deterioration rate (R), deterioration index and performance index, using the test results on weight loss, compressive strength, pulse velocity, and dynamic modulus of elasticity.

$$\begin{aligned}
 1) \text{ Deterioration rate} &= \frac{\% \text{ retention of impregnated concrete}}{\% \text{ retention of unimpregnated concrete}} \\
 &= R \quad (1.1)
 \end{aligned}$$

R_w is the deterioration rate of the parameter W (weight). Similarly, suffix s for compressive strength, v for pulse velocity and e for dynamic modulus of elasticity were used. These rates indicate the relative rate of deterioration for PIC as compared to that of UIC according to each

measurement. Deterioration index is so combined **that** if one material consistently gives poor performance over the other in all tests, the difference is exaggerated. Therefore the deterioration index is the product of all the four deterioration rates. The performance index combines the relative rates of deterioration (based on two measurements) with a comparison of the strength of the materials (based on two different measurements) after exposure.

$$\text{Performance Index} = R_w R_v \frac{S_i E_i}{S_u E_u} \quad (1.2)$$

where S is for strength, E is for dynamic modulus of elasticity, and subscript i for impregnated concrete and u for unimpregnated concrete.

Hiroshi Seki [2] started with deterioration index (D_f) as :

$$D_f = f(M_a, M_i, M_e, P_c, A, N, S, F)$$

and reduced to

$$D_f = M_a N S F \frac{PA}{G} \quad (1.3)$$

with few assumptions.

where M_a , M_i and P_c for quality and quantity of cement and aggregates used

M_e - factor regarding method of mixing, casting and curing of concrete

A - age of concrete;

N, S and F -- natural conditions relating to temperature, nature of exposure and freezing and thawing

P = water cement ratio, and

G = unit weight of cement.

Since the data is inadequate for estimating the coefficients, another equation given below, is suggested which incorporates the observed field data.

$$D_f = \frac{1}{2} \left[\frac{(\text{field observation} + \text{average depth of neutralization})}{2} + \frac{(\text{specific weight} + \text{coefficient of water absorption})}{3} + \frac{\text{concrete strength}}{3} \right] \quad (1.4)$$

He found that the reciprocal of D_f has nearly linear relationship with cement water ratio, means that cement-water ratio mainly influences the deterioration of concrete.

$$D_f = K P \quad \text{where } K \text{ is a constant} \quad (1.5)$$

According to Browne and Baker [3] the design life should be more than $t_1(x) + t_2(x)$, where x is a point of significance, t_1 is the time taken for the penetration of marine environment to point of interest within the concrete, and t_2 is the time taken for a material significance to the structure to take place at the point. Bazant [4,5] started with Nernst's equation for determining the electrode potential ($\Delta\phi$). This electrode potential is used in

estimating the rate of rust production per unit area of plane at $x = L$ (j_r), where L is cover thickness. If critical time (t_{cr}) at which corrosion would produce cracks through the whole cover is:

$$t_{cr} = t_p + t_{cor} \quad (1.6)$$

where t_p - time of depassivation and t_{cor} - duration of steady state corrosion.

$$t_p = \frac{1}{12 C_c} \left(\frac{L}{1 - \sqrt{u_c^*/u_c^S}} \right)^2$$

$$t_{cor} = \frac{f_{cor}^2}{S j_r} \frac{D \cdot \Delta D}{S j_r}$$

where C_c - coefficient of diffusion of chloride ions
 u_c^* - critical chloride ion concentration
 u_c^S - concentration of chloride ions in pores of concrete at the surface

$$f_{cor}^2 = 3.6 \text{ gm/cm}^2$$

D - diameter of the bar
 ΔD - increase in diameter of the bar due to rust and
 S - spacing of bars.

Browne [25] suggests an equation of the type given below for estimating the depth of penetration of chloride ion into the concrete in a given time

$$C_x(t) = C_o \left[1 - \operatorname{erf} \left(\frac{x}{2\sqrt{D_c t}} \right) \right] \quad (1.7)$$

where $C_x(t)$ - the chloride level at distance x after time t , for an equilibrium chloride level C_o at the surface

D_c - the chloride diffusion coefficient and
 erf - error function.

If representative values of D_c and C_o for different site conditions and concrete mixes are known, the cover required to prevent activation of the reinforcement for a specific life can be determined. Sinzo Mishiabashi et al. [26] suggested two seawater resistance factors:

$$\text{SDF}(1) = np / m \quad (1.8)$$

$$\text{SDF}(2) = (np / lm) 100 \quad (1.9)$$

where P - relative dynamic modulus of elasticity = RE_D

m = 200 cycles

n - number of cycles at RE_D of 60% (200 cycles, if $RE_D < 60\%$) and

l - length change .

Pranesh and Sudarsan [27] have proposed a mathematical model of the form

$$\frac{dc}{dt} = b_o + \sum_{i=1}^5 b_i x_i \quad (1.10)$$

where $\frac{dc}{dt}$ = corrosion rate, b_0 and b_i are regression coefficients and x_i are parameters involved in the corrosion phenomenon namely 1) duration of exposure of the material (days), 2) oxygen content (ml/l), 3) temperature ($^{\circ}\text{C}$), 4) velocity (m/sec) and 5) salinity (ppt) respectively.

1.2.7 Probabilistic Model for Strength

The statistical data on the ultimate compressive strength of concrete is reported by Jullian [28]. Chandrasekhar [29] has analyzed number of sets of field data and found that the probability of failure of concrete cube is about 0.12 for the strength ranging from 35 N/mm^2 to 50 N/mm^2 and the individual variation of the strength from the mean value is $\pm 10\%$. A coefficient of variation of 10% for excellent control, 15% for good control and 20% for poor control [30] can be taken for compressive strength of concrete. It is also suggested that normal distribution can be assumed, though the strength fits better in lognormal when the coefficient of variation is higher (15-20%). Mirza and MacGregor [31] have reviewed the variations in the location of reinforcing bars in slabs, beams and columns and found that normal distribution fits the data. They analyzed the results of about 4000 published and unpublished test results on deformed reinforced bars [32]. A coefficient

of variation of 10.7% for grade 40, 9.3% for grade 60 bars in mill test yield strength and 13.4% in static yield strength is reported by them.

Zsutty [33] presented a prediction equation for ultimate moment of under-reinforced concrete beams and discussed the sensitivity of the equation to various beam properties. Preliminary values for the coefficient of variation for in-place properties are given. This prediction equation allows inclusion of workmanship error in the variance of the strength prediction. Probability distribution for the strength of reinforced concrete beams with and without compression reinforcement is presented by Castello and Chu [34]. The failure probability depends upon the type of distributions observed for strength and stress. This sensitivity has been numerically illustrated by Jorgensen [35] and Ang and Amin [36, 37]. To avoid this sensitivity problem, number of engineering solutions have been proposed for determining the safety factors. Different people have used different criteria. Cornell [38] has considered only mean values and variances of the random variables without explicitly stating the failure probability. Cornell [38] and Rosenblueth [39] have prescribed an exponential relationship between the failure probability and the normalized safety index. Ang [36] specifies a

probabilistic treatment of statistical variabilities of information and handling of a problem through judgement factors where the information is lacking.

1.2.8 Structural Safety

Number of researchers have worked on the concept of safety factor or safety index, using the probability of failure which indicates the expected variations of stress and strength. Freudenthal [40] in 1956 presented the procedure for the numerical evaluation of factor of safety. Similarly, Pugsley [41] in 1957, Asplund [42] in 1958, Brown [43] in 1960, Brooding et al. [44] in 1964, Freudenthal et al. [45] in 1966, Cornell [46] in 1967, Ang and Amin [47] in 1968, Cornell [38], in 1969, Lind [48] in 1971, Ang and Cornell [49] in 1974, Sengupta and Dayaratnam [50] in 1979, and Ellingwood [51] in 1982 have worked on the concept of safety factor. Some authors [44, 45] have emphasized the need to correlate the safety factor with reliability or probability of failure. The second moment theory was developed by Cornell in 1969. The second moment safety index β is defined as [48, 52] central safety factor minus one divided by coefficient of variation of safety factor. Veneziano [53] claimed that β has number of limitations. So a new index γ , a function of Chebysheff upper-bound to probability of failure was proposed. Using γ , he has

made reliability comparisons between cases with the same β , but with different safety regions.

1.2.9 Probabilistic Design and Decision Theory

Benjamin [54] in 1968, Moses and Stevenson [55] in 1970, Ang [56] in 1973, Mayasandra et al. [57] in 1974, Ellingwood and Ang [58] in 1974, Ang and Cornell [49] in 1974, Lind [59] in 1977, and Ellingwood [60] in 1979 have worked on risk analysis and reliability based design. Moses and Kinser [61], Moses and Stevenson [55], Chandrasekhar [29], Murotsu et al. [62], Rosenblueth [63] and Rao [64] have considered the reliability as a constraint for optimizing the structural design. Allen [65] summarized the results of a probabilistic study of ultimate moment and ductility ratio of reinforced concrete members. He found that the expected ultimate moment in practice increases with the thickness of the member or higher the steel ratio. Monnier and Schmalz [66] discussed the probability of failure of a box beam. Using the statistical data, either derived from tests or assumed, the external moment and moment of resistance are generated by Monte Carlo simulation technique and the probability of failure is calculated.

Hiroyuki and Takeshi [67] have presented the method of estimating the reliability of deteriorating structures.

They have considered the resistance deterioration model and non-failure effect model. If a member has survived first load, then the corresponding left hand side tail of the probability distribution curve will be cut off. Similarly, as the member keeps on surviving the sequence of loads, correspondingly the left hand side tail of the distribution function also keeps on cutting off. They have also considered the case of estimating the failure probability if the future load distribution is known.

Sexsmith [63] worked on reliability based analysis of concrete structures. By assuming a generalized beta probability law for the under strength parameters, the parameters of the distributions were derived by making use of the experimental data through the Baye's rule. Benjamin [69, 54] Cornell [70] and Madsen and Lind [71] have investigated the application of Bayesian decision analysis to structural design.

1.3 Statement of the Problem

The present investigation is divided into two aspects. First one is on deterioration of concrete under sulphuric acid environment and to establish possible indices, which can be used in design. The second one is a study on probabilistic behaviour of exposed and unexposed beams.

Literature on both the aspects is reviewed . To the extent possible, literature available at this Institute in english language is reviewed. However much emphasis has been given for the work done during the last decade. Kawadkar [72] has done elaborate literature review on durability of concrete with more emphasis on the effect of marine environment on concrete. Freudenthal et al. [73] have reviewed the state of art on structural safety and analysis upto 1964. Task committee on structural safety [74] has given exhaustive literature covering upto 1972, which deals on the distribution of loads, strength, dynamic structural analysis and structural reliability analysis and design aspects.

The chemistry of cement, formation of cement compounds, hydration of cement and heat of hydration are reviewed. The effect of salts and acids on hydrated compounds are discussed. The stoichiometrical relations of some salts and acids with hydrated compounds are also presented. Using those stoichiometrical relations, an estimate of the amount of a particular chemical required for saturated reaction with one kiloNewton of cement is done.

An experimental investigation was carried out to estimate the deterioration of concrete exposed to sulphuric

acid under different concentrations. Concrete with aggregate cement ratio 4.5 , water cement ratio 0.45 and water cured for different ages were stored in sulphuric acid solution of 0.1 % , 1% and 5% concentrations. Constant concentration was maintained by adding the depleted amount of acid at regular and shorter intervals. The acid consumed by concrete with time was measured. Changes in weight and strength of acid exposed concrete, water saturated concrete and air dried concrete were measured at regular intervals. Depth of deterioration of concrete was also measured. Equations for estimating the amount of acid consumed in a given time and concentration, strength reduction factor, weight reduction factor, depth of penetration of acid, coefficient of variation (COV) of strength are established.

The rate of decrease of moment of resistance with the amount of acid consumed was determined. This moment of resistance is normalized with the moment of resistance of unexposed beam with same properties. This is studied for f_y 250, 415 and 500 N/mm^2 and the corresponding maximum percentage of tension reinforcement. If the coefficient of variation of individual variables is specified, the procedure for estimating the coefficient of variation of moment of resistance is presented. Figures are drawn to obtain the

ratio of moment of resistance to external moment for a specified probability of failure and variations in external moment and moment of resistance. When the ratio of moment of resistance to external moment is calculated for a given statistical variation of loadings and strength and preassigned probability of failure, it is rational to proportion the member without considering the partial safety factors for strengths and load factor for external moment. So the design procedure and charts are presented in Appendix B. The charts cover the range f_y 250, 415 and 500 N/mm² and f_{ck} 20 N/mm². The minimum cover requirement is checked using the equation for depth of penetration of acid. Similarly, life of an existing member when exposed to sulphuric acid is estimated from cover thickness point of view. The variation of cost of beam with probability of failure is studied through an example.

A procedure for estimating the revised probability of failure of a member using the data that it has survived for some years with probability of failure zero, is presented. Four examples are considered. The beam is subjected to probabilistic load and probabilistic strength and designed to survive for 50 years. The four cases are: 1) the probability of failure is constant with time, 2) probability of failure is constant for the first 20 years, and then increases due to

sudden decrease in moment of resistance by 20% due to reduction in concrete strength or cracking of a beam. The probability of failure is constant with time beyond 20 year period, 3) case three is similar to case two except the load increases by 20% instead of reduction in moment of resistance, and 4) time dependent deterioration of the concrete is considered, which causes the continuous reduction in moment of resistance. So there will be an increase in probability of failure every year. A coefficient of variation of 0.1 for external moment, 0.1 for strength of concrete and 0.05 for yield strength of steel are used. In case of strength deterioration of concrete the coefficient of variation is calculated for each case separately. The effect of survival for the first 10 years, 20 years, 30 years and 40 years period on the cumulative probability of failure at 50 years is studied through illustrative examples. The results are tabulated. The hazard rate, reliability and cumulative probability of failure of a deteriorating member are calculated for a beam taken up in case four.

CHAPTER II

KINETICS OF REACTIONS

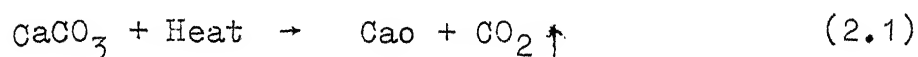
2.1 Introduction

The chapter presents chemistry relevant to cement, hydration of cement paste and effect of various chemicals on hydrated compounds. Deterioration of strength of concrete due to chemical attack is one of the factors affecting the durability of concrete. Attack of acids and salts on hydrated cement paste and corrosion of reinforced steel are the major factors which contribute to the deterioration. The influence of alkali - aggregate or alkali - silicate or alkali - carbonate reaction is relatively less significant in most of the concretes, unless otherwise the use of reactive type of aggregates in concrete can not be avoided. The raw materials for cement making, extraction of various oxides from the raw materials, compound formation, estimation of compound composition using Bogue's equations and Russian method and hydration of cement compounds are also discussed. The stoichiometrical relations of various chemicals with the hydrated cement compounds are presented. A discussion on mechanism of reinforced steel is also detailed.

2.2 Basic Materials and Products of Cement

Cement is made from chalk or lime stone and clay or shale. These two contain lime, silica and alumina, which are the essential ingredients of portland cement. The major chemical constituents of cement are lime (CaO), silica (SiO_2), alumina (Al_2O_3), iron oxide and gypsum ($\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$). Magnesia (MgO) and alkalis of low percent are also present [75] .

Lime stone (CaCO_3) when heated to about 1000°C , drives off carbon dioxide and forms calcium oxide [76] .



Silicon dioxide or silica is derived from clay or shale. It is also available in nature in the pure crystalline form as quartz and amorphous form as opal. Quartz is a stable form of silica. It takes transformations from low quartz to cristobalite, when heated upto 1470°C [15] . Alumina (Al_2O_3) which is also called as alumina sesquioxide, is available in the form of gybsite or hydrorgalite ($\text{Al}_2\text{O}_3 \cdot 3\text{H}_2\text{O}$), diaspor ($\text{Al}_2\text{O}_3 \cdot \text{H}_2\text{O}$) and boehmite or bauxite ($\text{Al}_2\text{O}_3 \cdot 2\text{H}_2\text{O}$). It also occurs combined with silica in clays [76] . The three forms of hydrated ferric oxide are α , β and γ [77] . β form has not been found in nature, but can be produced from ferric chloride solutions. The α and γ forms occur in nature as

goethite and lepidocrocite respectively. The most important iron ore is hematite (Fe_2O_3) and its hydrated form is limonite ($\text{Fe}_2\text{O}_3 \cdot \text{H}_2\text{O}$).

Gypsum, a hydrated calcium sulphate ($\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$), is naturally available. If dehydrated at 205°C it becomes anhydrite, which also occurs in nature [76]. The anhydrite is added to cement as a retarder to control the time of set of cement. In chemical analysis of cement this is reflected as sulphuric anhydrite (SO_3). Other retarders are, excess of $\text{Ca}(\text{OH})_2$ or mixture of $\text{Ca}(\text{OH})_2$ and CaSO_4 which in fact acts as a very good retarder, ~~calcium sulphate~~, or calcium sulphate or calcium iodide or sodium carbonate and sodium silicate solutions. Ferrous oxide (FeO) is also present in portland cement ranging from traces to 0.4% [15].

Magnesia or periclase is derived from magnesium carbonate. Mineral magnesite (MgCO_3) and mineral dolomite ($\text{CaMg}(\text{CO}_3)_2$ or $\text{CaCO}_3 \cdot \text{MgCO}_3$) occur naturally. The magnesia is found to react with aggregates in concrete and causes disintegration, and hence affects the strength of concrete [15]. So the magnesia content in a finished cement is limited to a specified value. The IS 269 - 1967 specifies a maximum of six percent in ordinary, rapid hardening and low heat cements. IS 455 - 1967 suggests a maximum of eight percent for blast-furnace slag cement. IS 1489 - 1967

specifies six percent for portland pozzolana cement and IS 6909 - 1973 specifies a maximum of ten percent for super-sulphate cement. Nothing has been specified for high alumina cement.

Nothing is known about the reactions of the alkali compounds in cement making and cement hydration process, even though Na_2O and K_2O are often present in cement in very small quantities.

2.3 Formation of Compound of Cement

When lime and silica are heated together, four distinct compounds can form : 1) metasilicate($\text{CaO} \cdot \text{SiO}_2$), 2) the compound ($3\text{CaO} \cdot 2\text{SiO}_2$), 3) orthosilicate($2\text{CaO} \cdot \text{SiO}_2$) and 4) the compound $3\text{CaO} \cdot \text{SiO}_2$. Out of these four compounds, only orthosilicate and compound $3\text{CaO} \cdot \text{SiO}_2$ are present in cements. Orthosilicate exists in four forms: α . $2\text{CaO} \cdot \text{SiO}_2$, α' $2\text{CaO} \cdot \text{SiO}_2$, β $2\text{CaO} \cdot \text{SiO}_2$ and γ $2\text{CaO} \cdot \text{SiO}_2$. α $2\text{CaO} \cdot \text{SiO}_2$ is stable above $1420^\circ - 1447^\circ\text{C}$ and changes reversibly to α' $2\text{CaO} \cdot \text{SiO}_2$ form on cooling. α' $2\text{CaO} \cdot \text{SiO}_2$ is stable at $800^\circ - 1447^\circ\text{C}$ and inverts to β form at $650^\circ - 675^\circ\text{C}$. β $2\text{CaO} \cdot \text{SiO}_2$ further inverts to γ $2\text{CaO} \cdot \text{SiO}_2$ at 675°C . But at the rates of cooling of commercial cements β form is preserved in the clinker, and it forms round grains or crystals with polysynthetic twinning. The pure compound of tricalcium

silicate ($3\text{CaO} \cdot \text{SiO}_2$) is formed by a solid state reaction between CaO and $2\text{CaO} \cdot \text{SiO}_2$ above 1250°C and below 1900°C . If the cooling rate is not too slow, tricalcium silicate remains unchanged and is stable at ordinary temperature. Above 1900°C it melts incongruently to CaO and liquid [78] .

There are four stable compounds in $\text{CaO} \cdot \text{Al}_2\text{O}_3$ system: $3\text{CaO} \cdot \text{Al}_2\text{O}_3$, $\text{CaO} \cdot \text{Al}_2\text{O}_3$, $\text{CaO} \cdot 2\text{Al}_2\text{O}_3$ and $\text{CaO} \cdot 6\text{Al}_2\text{O}_3$. Out of these four, only $3\text{CaO} \cdot \text{Al}_2\text{O}_3$ is present in portland cement and rest all are present rarely or as a main constituent in high alumina cement. Tetracalciumaluminoferrite (C_4AF) is a solid solution ranging from C_2F to $\text{C}_6\text{A}_2\text{F}$. So it is not a compound but typifies the composition of a solid solution [15].

Figure 2.1 illustrates the formation of compounds at various temperatures. The compounds are formed within the temperature range of $800^\circ - 1450^\circ\text{C}$. Major part of $3\text{CaO} \cdot \text{Al}_2\text{O}_3$, $4\text{CaO} \cdot \text{Al}_2\text{O}_3 \cdot \text{Fe}_2\text{O}_3$ and $2\text{CaO} \cdot \text{SiO}_2$ form in the temperature range of $1100^\circ - 1200^\circ\text{C}$. Only $3\text{CaO} \cdot \text{SiO}_2$ forms in the range of $1200^\circ - 1450^\circ\text{C}$.

2.4 Estimation of Compounds of Cement

The compounds $3\text{CaO} \cdot \text{SiO}_2 (\text{C}_3\text{S})$, $2\text{CaO} \cdot \text{SiO}_2 (\text{C}_2\text{S})$, $3\text{CaO} \cdot \text{Al}_2\text{O}_3 (\text{C}_3\text{A})$ and $4\text{CaO} \cdot \text{Al}_2\text{O}_3 \cdot \text{Fe}_2\text{O}_3 (\text{C}_4\text{AF})$ can be estimated from the quantities obtained in oxide analysis of cement, using Bogue's [15] equations and Russian method [79] .

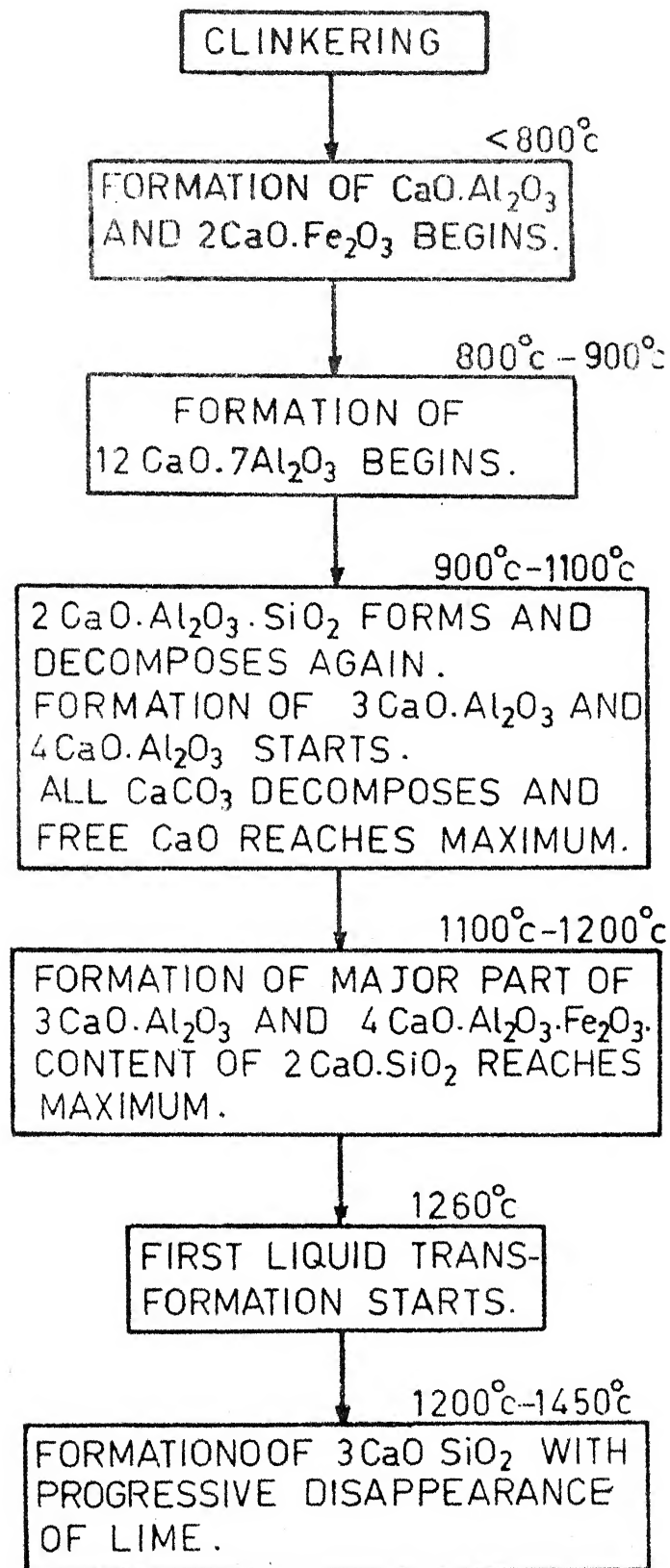


FIG. 2.1 BLOCK DIAGRAM OF COMPOUND FORMATION AT VARIOUS TEMPERATURES [15]

Bogue's Equations

$$C_3S = 4.071 CaO - 7.6 SiO_2 - 6.718 Al_2O_3 - 1.43 Fe_2O_3 - 2.852 SO_3 \quad (2.2)$$

$$C_2S = 2.867 SiO_2 - 0.754 C_3S \quad (2.3)$$

$$C_3A = 2.650 Al_2O_3 - 1.692 Fe_2O_3 \quad (2.4)$$

$$C_4AF = 3.043 Fe_2O_3 \quad (2.5)$$

As per ASTM C150, the above equations are recommended for Al_2O_3/Fe_2O_3 ratio greater than or equal to 0.64. If the ratio is less than 0.64, C_3A does not exist in cement and calcium aluminoferrite solid solution ($C_4AF + C_2F$) may form. So in the modified equation for C_4AF , the presence of C_2F is included.

$$C_3S = 4.071 CaO - 7.6 SiO_2 - 4.479 Al_2O_3 - 2.359 Fe_2O_3 - 2.852 S \quad (2.6)$$

$$C_2S = 2.867 SiO_2 - 0.7544 C_3S \quad (2.7)$$

$$C_3A = 0 \quad (2.8)$$

$$C_4AF = 2.10 Al_2O_3 + 1.702 Fe_2O_3 \quad (2.9)$$

Russian Method

$$KS_K = \frac{CaO - (1.65 Al_2O_3 + 0.35 Fe_2O_3)}{2.8 SiO_2} \quad (2.10)$$

$$C_3S = 3.8 SiO_2 (3KS_K - 2) \quad (2.11)$$

$$C_2S = 8.6 SiO_2 (1-KS_K) \quad (2.12)$$

$$C_3A = 2.65 (Al_2O_3 + 0.64 Fe_2O_3) \quad (2.13)$$

$$C_4AF = 3.04 Fe_2O_3 \quad (2.14)$$

Ranges of compound composition for ordinary and rapid hardening portland cements [80] are :

Ordinary Portland Cement (avg. of 21 samples):

C_3S	42 - 67 % with an average of 49 %
C_2S	8 - 31 % with an average of 25 %
C_3A	5 - 14 % with an average of 12 %
C_4AF	6 - 12 % with an average of 8 %
$CaSO_4$	2.6 - 3.4 % with an average of 2.9 %

Rapid Hardening Portland Cement (Avg. of 5 samples):

C_3S	34 - 70 % with an average of 56 %
C_2S	0 - 38 % with an average of 15 %
C_3A	7 - 17 % with an average of 12 %
C_4AF	6 - 10 % with an average of 8 %
$CaSO_4$	2.2 - 4.6 % with an average of 3.9 %

TABLE 2.1 : TYPICAL OXIDE COMPOSITION OF CEMENT [81]

Oxide	CaO	SiO ₂	Al ₂ O ₃	Fe ₂ O ₃	MgO	K ₂ O	Na ₂ O	SO ₃
Weight %	63	22	6	2.5	2.6	0.6	0.3	2.0

Using equations(2.2)to(2.14) and the data in Table 2.1 compound composition of the above cement can be calculated. They are tabulated in Table 2.2.

TABLE 2.2 : COMPOUND COMPOSITION FOR THE CEMENT GIVEN IN TABLE 2.1

Compound	Method	
	Bogue [*]	Russian ⁺
C_3S	40 %	46 %
C_2S	33 %	28 %
C_3A	12 %	12 %
C_4AF	8 %	8 %

* $Al_2O_3/Fe_2O_3 > 0.64$, hence equations (2.2) - (2.5) were used.

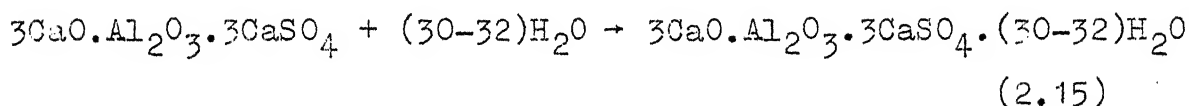
+ $KS_K = 0.85$

2.5 Hydration of Cement

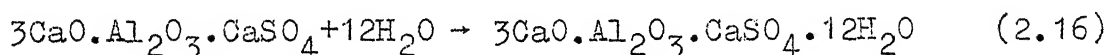
If water is added to cement many complex chemical changes take place. Lechatlier [9] about 100 years ago, was first to observe that the hydration of individual compounds of cement is chemically same as that of the hydration of cement under similar conditions. Later, Steinour[82] and Bogue et al.[83] confirmed the above observation.

The rate of hydration of the compounds in the early stages follows $C_3A > C_3S > C_4AF > C_2S$. C_3S reacts very fast. Gypsum or any other retarder is added to control the early reactions of C_3A and retard the process of hardening.

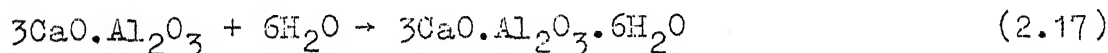
Otherwise, without any retarder, C_3A reacts fast and flash set would occur leaving the product unusable. It has been observed that ettringite (high sulphate form) or monosulphoaluminate (low sulphate form) or hydrogarnet occurs for different values of $CaSO_4 \cdot 2H_2O / C_3A$ molar ratios [81]. If the above molar ratio is 3.0 ettringite forms:



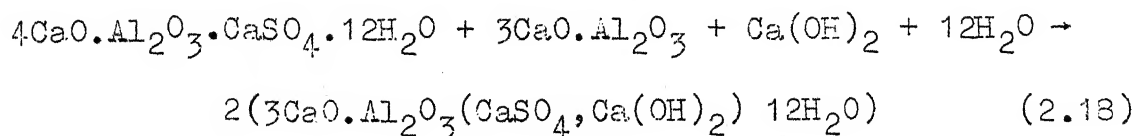
If the ratio is 1.0 monosulphoaluminate forms:



If the ratio is 0 hydrogarnet forms:



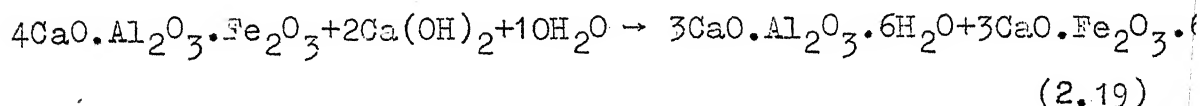
If SO_3 is used up early in the course of reactions, the ettringite starts reacting further with C_3A and transforms to low sulphate or monosulphate form. In such cases a solid solution between $4CaO \cdot Al_2O_3 \cdot CaSO_4 \cdot 12H_2O$ and $4CaO \cdot Al_2O_3 \cdot 13H_2O$ forms:



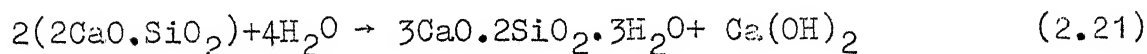
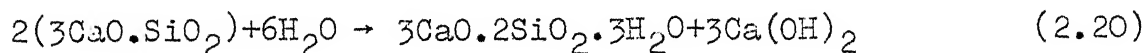
With time the calcium sulphoaluminate converts to crystals of $4\text{CaO} \cdot \text{Al}_2\text{O}_3 \cdot 12\text{H}_2\text{O}$ [84] first and later changes to the cubic form $3\text{CaO} \cdot \text{Al}_2\text{O}_3 \cdot 6\text{H}_2\text{O}$ (hydrogarnet). Hydrogarnet is a probable stable form of calcium aluminate hydrate, which exists in hydrated cement paste [9].

When monosulphoaluminate brought into contact with sulphate ions, ettringite will form. This is the basis for sulphate attack of concretes when exposed to an external supply of sulphate ions.

The C_4AF contributes little to any significant behaviour of cement. The straight forward hydration of C_4AF is in doubt, but the following has been suggested by Illston et al. [75] .



The hydration of dicalcium silicate and tricalcium silicate are as follows:



The final hydrated form of both the silicates is same except in the formation of $\text{Ca}(\text{OH})_2$. If the same quantity of C_3S and C_2S are hydrated, the lime produced in the hydration of C_3S is about twice as much as produced in C_2S . C_3S reacts

fast and contributes for initial strength development. C_2S reacts little slow and contributes for later age strength development. The calcium silicate hydrate ($3CaO \cdot 2SiO_2 \cdot 3H_2O$) forms the bulk of the hydrated compounds. This is also known as tobermite gel. During the formation of gel, slight increase in volume takes place, which is directly proportional to the development of strength at all ages. Due to liberation of $Ca(OH)_2$ during hydration, the P^H of setting cement increases [75] .

2.6 Heat of Hydration of Cement

During the hydration of cement compounds, heat upto 120 cal/gm (502 J/gm) is liberated. This is exothermic like many other chemical reactions. Lerch and Bogue [85] reported the heat of hydration of pure compounds as:

TABLE 2.3 : HEAT OF HYDRATION OF CEMENT COMPOUNDS

C_3S	C_2S	C_3A	C_4AF	
120	62	207	100	cal/gm
502	259	866	418	J/gm

The heat of hydration of cement varies with curing temperature, compound composition and age. Lerch and Ford [86] found the heat of hydration of cement developed

after 72 hours at different temperatures: 12 samples of type I were tested. The heat evolved was 36.9 cal/gm (155 J/gm) at 4°C, 68.0 cal/gm (284 J/gm) at 24°C, 73.9 cal/gm (309 J/gm) at 32°C, 80.0 cal/gm (335 J/gm) at 41°C.

Similarly 4 samples of type III were tested. The heat evolved was 52.9 cal/gm (221 J/gm) at 4°C, 83.2 cal/gm (348 J/gm) at 24°C, 85.2 cal/gm (357 J/gm) at 32°C and 93.2 cal/gm (390 J/gm) at 41°C. An equation was proposed by Woods et al. [87] of the type given below for calculating the heat of hydration of 1gm cement.

$$136(C_3S) + 62(C_2S) + 200(C_3A) + 30(C_4AF) \quad (2.22)$$

The figures in brackets denote the percentage by weight of the individual compounds present in cement. Sidney Mindess et al. [81] proposed equations for the heat evolved at three days and one year in J/gm of cement as given below.

$$H_{3 \text{ days}} = 240(C_3S) + 50(C_2S) + 880(C_3A) + 290(C_4AF) \quad (2.23)$$

$$H_{1 \text{ year}} = 490(C_3S) + 225(C_2S) + 1160(C_3A) + 375(C_4AF) \quad (2.24)$$

Verbeck and Foster [88] have shown the variation of average heat of hydration with age graphically. Consistently the heat of hydration increases in the order type IV, type V, type II, type I and type III at all ages.

2.7 Corrosion of Concrete

Corrosion may be defined as the destructive attack of external aggressive media, which affects the durability of concrete. The various factors which cause the destructive actions are grouped into three categories:

- 1) Physical factors like, temperature variations, freezing and thawing, wetting and drying, wear and abrasion etc.
- 2) Chemical factors like acids (dissolving attack), salts (swelling attack), corrosion of reinforced steel, alkali-aggregate reaction etc. and
- 3) Biological factors like, microorganisms, fungi, algae, marine animals, insects etc.

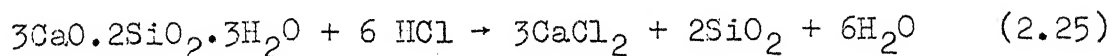
The factors may act independently or in combination. The chemical factors are the important factors, which cause accelerated deterioration of plain, reinforced and prestressed concrete. Deterioration of concrete may take place due to corrosion of hardened cement paste in concrete or due to corrosion of reinforced steel or due to alkali aggregate or alkali-silicate or alkali-carbonate reaction.

Theoretically only about 30% of water by weight of cement is required for complete hydration of cement compounds. The excess water added will collect in the capillary pores of concrete as a solution of calcium hydroxide. If the water

cement ratio is more, the concrete is more porous and hence the deterioration is also more.

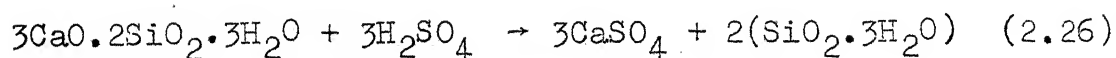
The deterioration of concrete due to chemical corrosion can be either dissolving attack or swelling attack. Strong mineral acids like hydrochloric acid, sulphuric acid and nitric acid dissolve all the components of set cement. Weak acids like carbonic acid forms water soluble compounds, so the attack is at slower rate and takes very long time for considerable damage. Organic acids like lactic acid, acetic acid or citric acid softens the concrete. DIN 4030 [89] specifies a range of P^H values for different degrees of acid attack. P^H value less than 4.5 has a very strong attack, between 4.5 and 5.5 a strong attack and between 5.5 - 6.5 a weak attack.

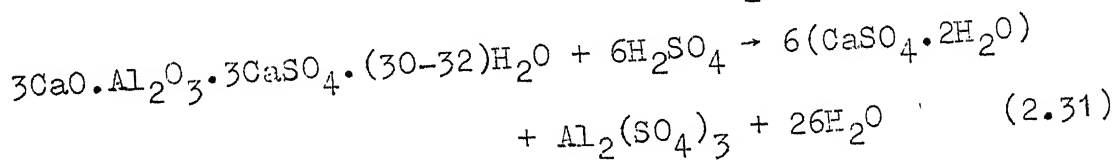
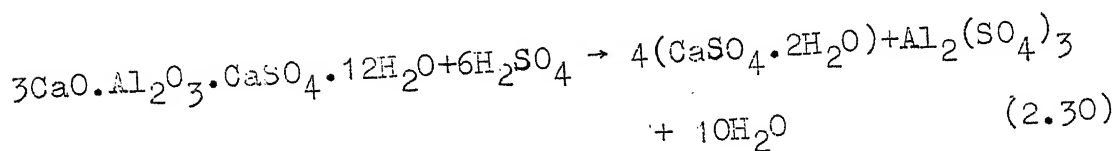
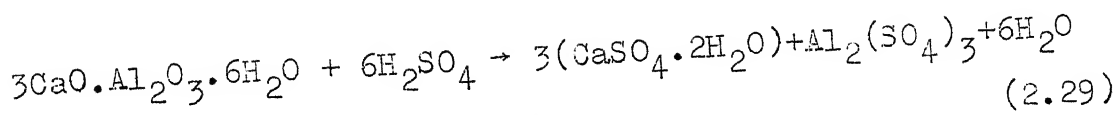
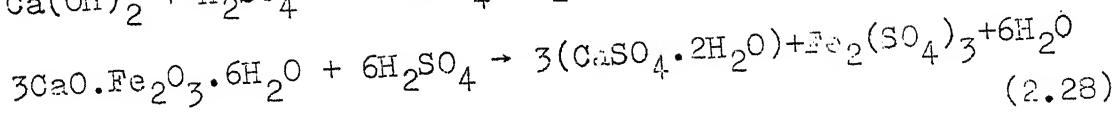
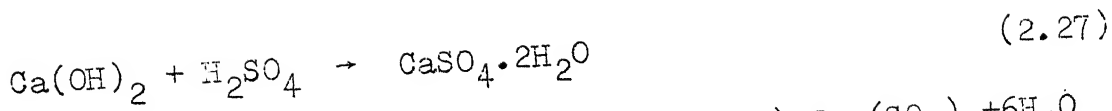
If strong acids act on cement paste, calcium, aluminium, and iron salts and silica gel will form [89]. The typical reaction of hydrochloric acid on tricalcium disilicate hydrate is:



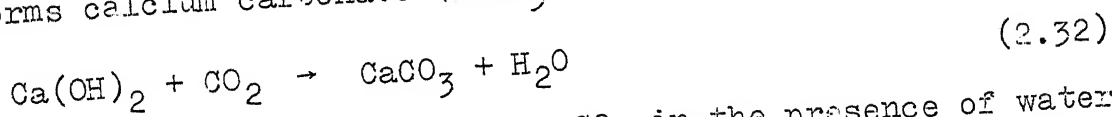
Calcium chloride thus formed is highly soluble.

The reactions of sulphuric acid with different hydrated cement compounds are:

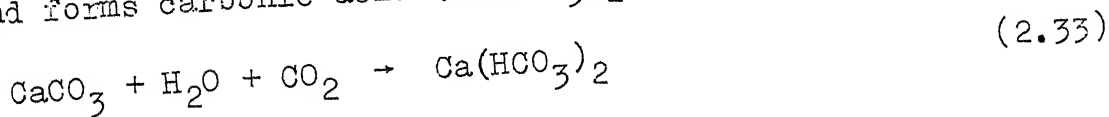




Calcium hydroxide reacts with carbon dioxide and forms calcium carbonate (CaCO_3) known as carbonation.



This CaCO_3 further reacts with CO_2 in the presence of water and forms carbonic acid ($\text{Ca}(\text{HCO}_3)_2$).



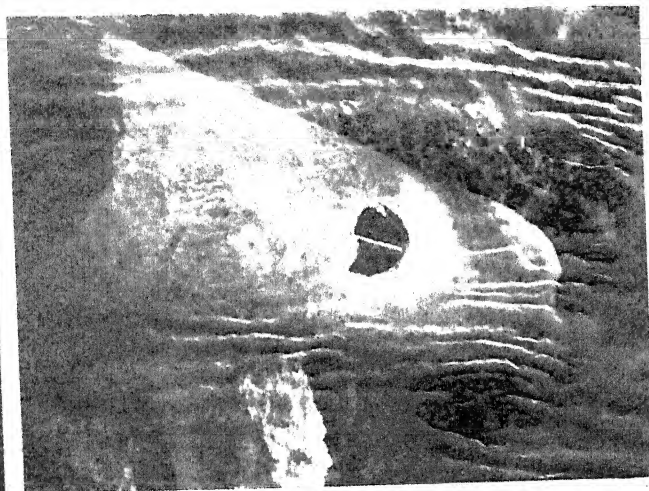
CaCO_3 is sparingly soluble, whereas $\text{Ca}(\text{HCO}_3)_2$ is highly soluble. So it is capable of dissolving lime from concrete. Both in the process of carbonation and in the formation of carbonic acid, the lime is dissolved. The alkalinity of concrete is maintained by the presence of Ca(OH)_2 . So the reduction in lime reduces the P^{H} of concrete.

Hydrogen sulphide formed in effluents dissolves in water and forms a weak acid. This can be absorbed by the concrete in contact and oxidizes to form sulphurous acid and then by oxidation to sulphuric acid.

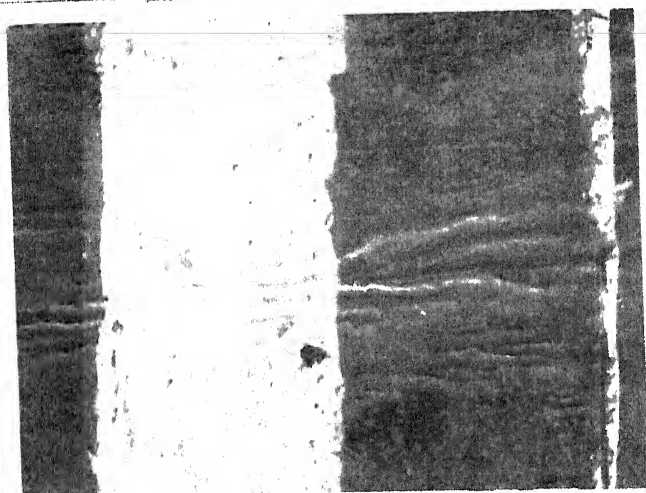
Sulphur dioxide gas from flue gases takes moisture and converts to sulphurous acid. By oxidation the sulphurous acid converts to sulphuric acid. When coal or coke is stored against the concrete surface, the sulphur contained in it oxidizes to sulphur dioxide, later converts to sulphuric acid. This causes the concrete to be attacked rapidly.

Figure 2.2 shows the damaged concrete sewer pipes removed after 18 years of use. The actual size of the pipe is 900 mm diameter and 75 mm thick. Figure 2.2a is less damaged compared to Figures 2.2b and 2.2c. It is observed that some of the pipes are split into two parts (Fig. 2.2b) along longitudinal bars, which can be seen on the right hand side edge in the Fig. 2.2b. Figure 2.2c is more damaged. The concrete surface is fully damaged and covered by the settlement of solids from sewage. The category of damage of most of the pipes lie in between Fig. 2.2b and Fig. 2.2c.

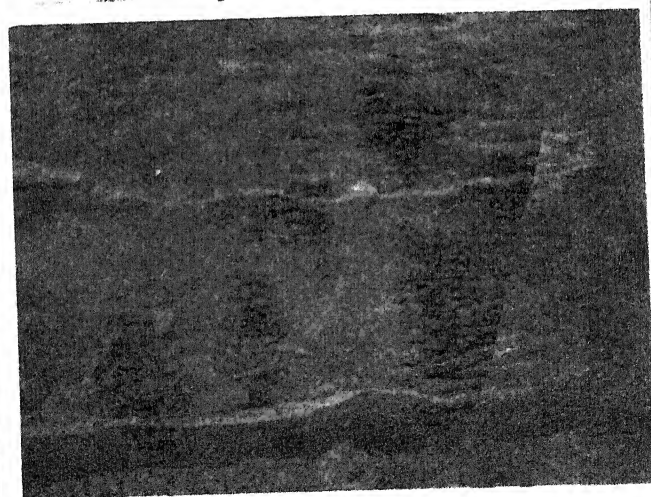
Seawater contains salts (chlorides and sulphates). Some clays contain alkali, magnesium or calcium sulphates and ground water in such type of clays is in effect a sulphate attack. The higher chloride content in seawater reacts with



(a)



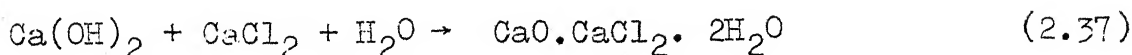
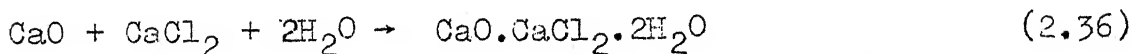
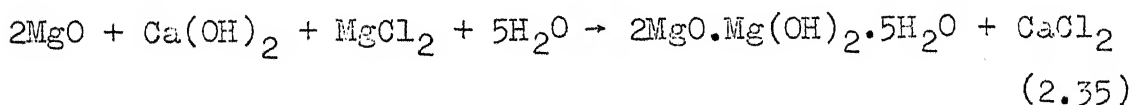
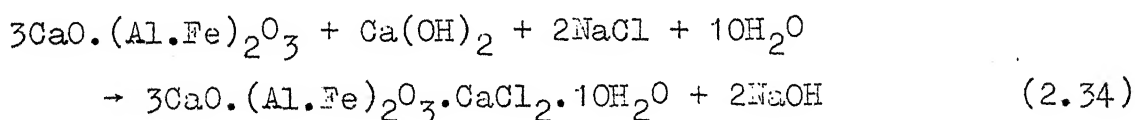
(b)



(c)

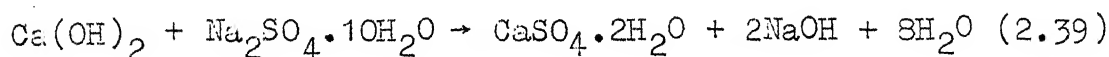
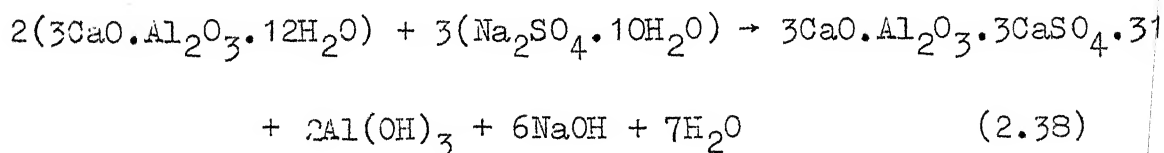
FIG. 2.2 : DAMAGED CONCRETE SEWER PIPES
(a) LESS DAMAGED , (b) SPLIT-UP LONGITUDINALLY
(c) SEVERLY DAMAGED

free lime and leaches it out in the form of calcium chloride. It is highly soluble and leads to the increase in porosity of concrete. Chlorides adversely affect the polymerization of the calcium silicates, hence hinder the long time strength development [90] . For the concentration of magnesium chloride present in seawater, the attack is minor [91] . Only strong solution of magnesium chloride has some effect on ordinary portland cement [15] . Magnesium ions are more aggressive than chloride ions. Magnesium chloride reacts with calcium hydroxide and forms magnesium hydroxide. This reduces the P^H of concrete. This forms a layer on the surface and prevents further diffusion of magnesium ions [92] . The stoichiometrical relations of various chlorides with hydrated cement compounds are [92] :



If the concrete is attacked by sulphates, highly insoluble and voluminous products than the compounds they replace, will form. This causes an increase in internal

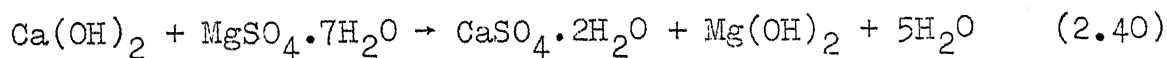
volume. As a result internal pressure is exerted leading to cracking and spalling of concrete. Due to cracking and spalling fresh area is exposed and deterioration accelerates. The two major sulphates that deteriorate the concrete are sodium sulphate and magnesium sulphate. Sulphates in solid form do not attack concrete, but reacts with set cement when they are in liquid form. Alternative wetting and drying further contributes to the deterioration. Sodium sulphate reacts with tricalcium aluminate hydrate and calcium hydroxide. When it reacts with tricalcium aluminate hydrate calcium sulphoaluminate (sulphoaluminate corrosion) forms, whose increase in volume is 227% in solid phase. Similarly the reaction with $\text{Ca}(\text{OH})_2$ forms gypsum (gypsum corrosion), whose increase in volume is 124% in solid phase [9]. The stoichiometrical relations of the above two reactions are:



The effect of sodium sulphate increases with increase in concentration upto 1.0 % and beyond 1.0 % the increase is at a very low rate [91] . Alkali sulphates do not react with

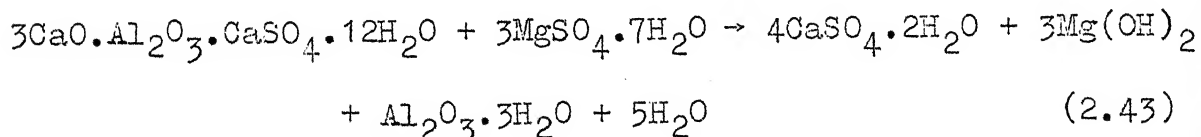
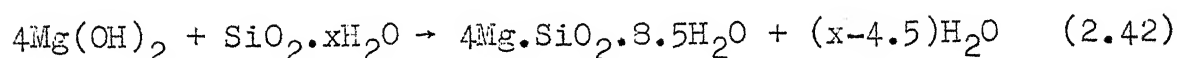
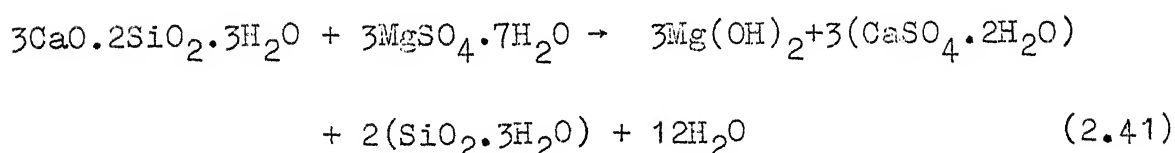
calcium silicate hydrate to any significant extent [93,15,19, 94,95] . Reason being, they are more insoluble than calcium sulphate and alkali silicates, which would occur. But Thorvaldson[96] and Thorvaldson et al. [97] observed that the mortar specimens made with pure C_3S and pure C_2S were adversely affected by Na_2SO_4 solution.

Magnesium sulphate not only causes destruction due to swelling but also softens the surface of the concrete. As discussed earlier magnesium sulphate does react with hydrated cement paste, only in liquid form. Upto 0.5 % concentration the rate of attack increases with increase in concentration. Beyond 0.5 % the rate of increase is smaller [91] . Magnesium sulphate not only reacts with lime and calcium aluminate hydrate but also reacts with calcium silicate hydrate. The reaction of magnesium sulphate with $Ca(OH)_2$ causes gypsum corrosion:

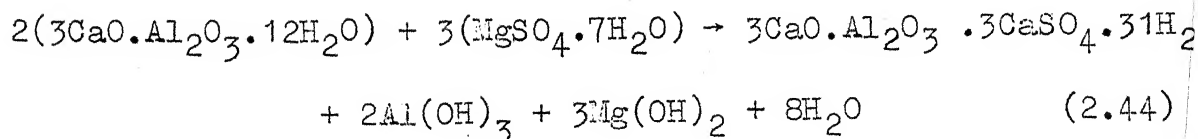


Gypsum thus formed crystallizes and is accompanied by increase in volume. The increase in volume is about 17.7% and that of magnesium hydroxide is about 20% [19]. The $Mg(OH)_2$ forms in the existing pores without causing any disruptive reaction [81] . Magnesium sulphate reacts with calcium silicate hydrate and produces magnesium hydroxide, gypsum and

silica gel. The magnesium hydroxide and silica gel further reacts and magnesium silicate forms. This magnesium silicate does not have binding power in contrast to silica gel. So it is reasonable to say that this is a final stage in the deterioration of concrete [98]. It reacts with monosulphoaluminate and produces, gypsum, magnesium hydroxide and alumina hydrate. The above process is entirely known as magnesium corrosion. The precipitation of $\text{Mg}(\text{OH})_2$ increases the rate of gypsum corrosion [81]. The stoichiometrical relations of magnesium corrosion are:



The magnesium sulphate reacts with calcium aluminate hydrate and ettringite forms (sulphoaluminate corrosion) [81] .



This crystallizes in the form of needle shaped crystals and during this process the component volume increases to about 227 % in solid phase, causing destruction of concrete. If MgSO_4 concentration is much higher sulphoaluminate corrosion is replaced by magnesium corrosion. Sulphate is expressed as a number of parts by weight of SO_3 per million (ppm). 1000ppm and 2000 ppm are considered to be moderately severe and severe respectively, specially if the magnesium sulphate is the predominant constituent [9] . As per DIN4030, 200 - 600 ppm corresponds to weak attack, 600 - 3000 ppm corresponds to strong attack and above 3000 ppm corresponds to very strong attack [89] . 0.2 % and 0.5 % are the corresponding values of soluble sulphates in soils [9] . DIN 4030 specifies 0.2% - 0.5 % for weak attack and above 0.5% for strong attack [89] .

Figures 2.3a - 2.3e show the effect of marine environment on reinforced concrete. Figure 2.3b and 2.3c are part of the electric pole (Fig.2.3a) on beach road. These are enlarged to emphasize the magnitude of damage caused to electric pole by marine environment, Similarly Figs.2.3d and 2.3e are the photographs of roof slab of a residential building near sea shore. This was constructed 20 years back. The first scalling was observed 15 years after construction, then it was plastered. The rescaling off

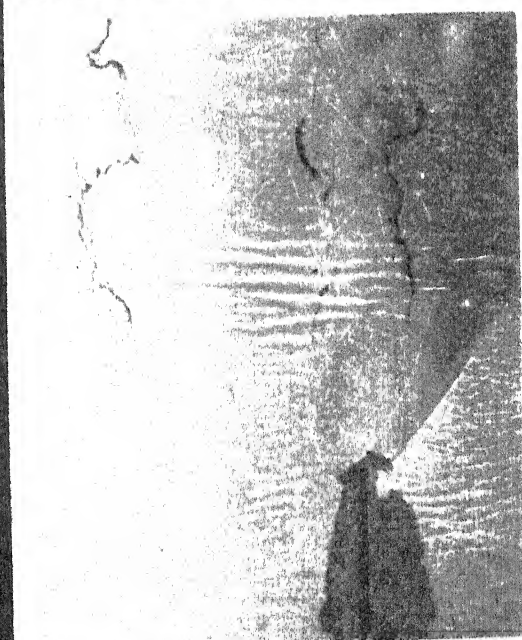
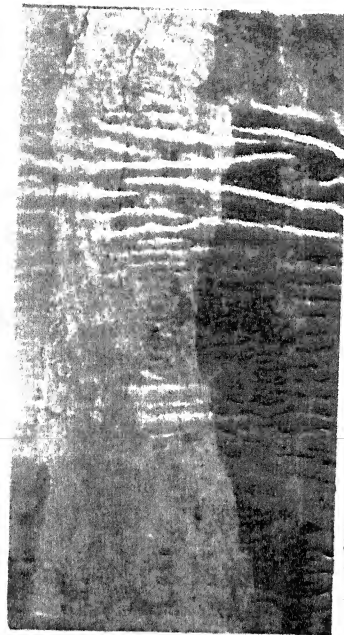
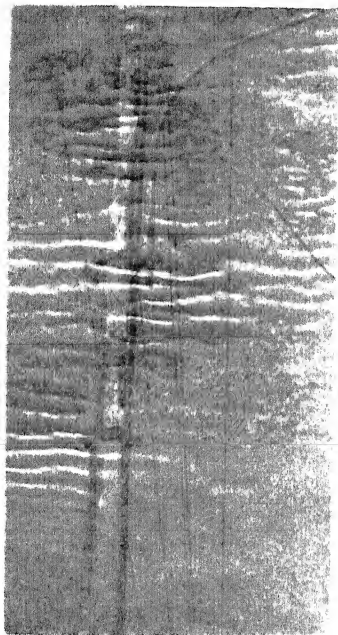


FIG.2.3 : DAMAGED ROOF SLAB AND ELECTRIC POLE NEAR SEASHORE
 (a) ELECTRIC POLE (b) and (c) ENLARGED PORTIONS
 OF THE ELECTRIC POLE (d) and (e) SCALLING OFF OF
 A ROOF SLAB.

the plastered one also can be seen in Fig. 2.3e . Table 2.4 summarizes the effect of some of the chemicals on hydrated cement paste.

TABLE 2.4 : REACTION OF SOME OF THE CHEMICALS WITH HYDRATED CEMENT COMPOUNDS AND THEIR EFFECT

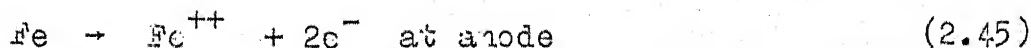
S.No.	Chemical	Cement Compound	Effect
1	Acids	Cement paste	Calcium, aluminium and iron salts and silica gel forms. Calcium chloride is highly soluble.
2	CO ₂	Ca(OH) ₂	Forms carbonic acid in the presence of CO ₂ and water, which is highly soluble. P ^H reduces.
3	Chlorides	Free lime calcium silicates	Lime leaches out, porosity increases, hinders polymerization of calcium silicate hydrate. P ^H reduces
		Ca(OH) ₂	P ^H reduces
4	Sulphates	Ca(OH) ₂ , Tricalcium aluminate hydrate	Forms gypsum, which increases the volume by 117.7%. Forms calcium sulphoaluminate, volume increases by 227%. Forms Mg(OH) ₂ volume increases by 120%.
	MgSO ₄ Na ₂ SO ₄		
	MgSO ₄	Tricalcium silicate hydrate	Forms Mg(OH) ₂ , silica gel, gypsum. Mg(OH) ₂ reacts with silica gel and forms magnesium silica gel, which has no binding power in contrast to the silica gel.

2.8 Corrosion of Reinforced Steel

Steel in concrete is passivated due to its high levels of alkalinity ($P^H > 12.5$) as explained in previous sections. This passivation can be prevented due to faults in concrete like tears, cracks, agglomeration of sand etc. If the concrete is porous or cracked, allows the ingress of atmospheric gases and water, which may convert calcium hydroxide to carbonic acid (Eq.(2.32) and Eq.(2.33)). In this process P^H of concrete reduces to some what below 9. If 0.3 % of CO_2 by volume present in air, allows carbonation [89]. Due to reduction in P^H , steel is prone to corrosion. The two laboratory methods for assessing the depth of carbonation have been X-ray diffraction method and chemical analysis method. In both the methods, the sample is collected at various depths of the specimen and is investigated with any one of the methods. The simplest method being, an acid based indicator should be sprayed on freshly broken surface of the concrete. On spraying the indicator, the uncarbonated layer, which is highly alkaline portion, changes its colour. If alizarin yellow R is used colour changes from yellow to orange red, with thymolphthalein from colourless to blue and with phenolphthalein from colourless to purple red [99]. R. Kondo and his Co-workers (1968) [91] have suggested that the compressive strength of concrete is inversely proportional to

carbonation or directly proportional to a ratio of porosity to carbon dioxide.

Basically rust formation is an electrochemical process. It is believed that [2,100] steel is passivated from corrosion by forming an oxide film on the surface (approximately $10,000 \text{ \AA}$ thick). This surface film is mainly a ferric oxide (Fe_2O_3). This can be depassivated by reducing the alkalinity of concrete. But other researchers [101, 102] observed that ferric oxide film might not exist to protect steel, but the formation of chloride ion (Cl^-) film indicates the corrosion. Contact between two dissimilar surfaces (steel is more electronegative than surrounding media), electrolytes such as acids, alkalis and sulphates and varying availability of oxygen, favours the conditions of inequality. Higher the potential difference, faster the rate of corrosion. Normally the anode areas develop on steel surface [90]. Iron is oxidized and iron ions pass into the solution at the anode surface.



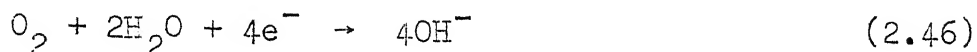
where e^- = electron.

This makes steel electrically negative and adjacent to anodic areas begin to function as cathode. At the cathodic area, the oxygen dissolved in pore water reacts with the

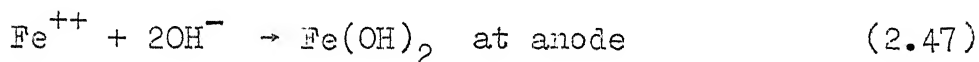
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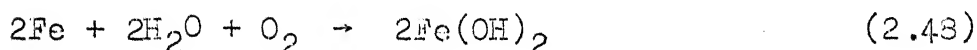
electrons coming from anode, in the presence of water to form hydroxyl ions (OH^-).



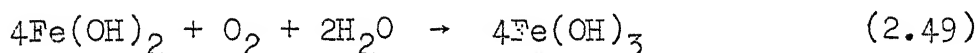
Electric current is created in steel due to liberation of electrons at cathode. The negative hydroxyl ions liberated at cathode are transferred to anode, electrically neutralizes the dissolved Fe^{++} , forming a solution of ferrous hydroxide.



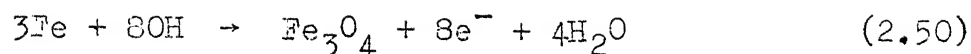
Now the total reaction is:



This ferrous hydroxide further reacts with the available oxygen and water and gives hydrated iron oxide.

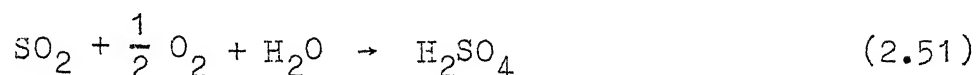


This constitutes hydrated red rust ($\text{FeO}(\text{OH}) + \text{H}_2\text{O}$), the final product, which precipitates from the solution. The volume of red rust is four times that of steel. There are other forms of rust: for example $\text{HFeO}(\text{OH})$, HFeO_2 , FeSO_4 and Fe_2O_4 (black rust). The black rust is formed in a continuous very deep water immersion where oxygen supply is restricted [4, 5]. In seawater the oxygen content varies from approximately 7 ppm at surface to 3 ppm at 100 m depth. So the black rust can form in offshore platforms etc. [3].

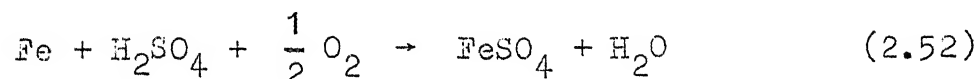


The volume of black rust is twice the volume of steel.

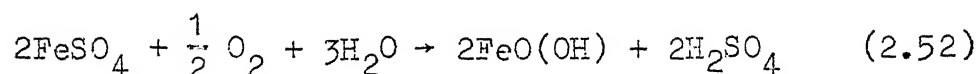
The impurities in atmosphere like sulphur dioxide and chlorides accelerate the process of corrosion even at low concentrations. With a suitable catalyst sulphur dioxide oxidizes to sulphuric acid.



Sulphuric acid reacts with iron in the presence of oxygen and forms ferrous sulphate (FeSO_4).



This ferrous sulphate is further oxidized in the presence of oxygen and produces rust and sulphuric acid



This sulphuric acid can further react with steel and form more rust [89] . The rust formed on the surface is porous and permits the ingress of atmospheric gases or other chemicals for further deterioration. The relative humidity between 65% and 90% contributes appreciably for corrosion of mild steel [91] .

Chloride ions present in the concrete depassivates the protective layer if a certain threshold value exceeds. Chlorides are capable of setting up differential concentration

CHAPTER III

EXPERIMENTAL INVESTIGATION

3.1 Introduction

The details of experimental investigation, relevant analysis, results and discussion are presented in this chapter. A concrete mix with aggregate cement ratio (A/C) of 4.5 and water cement ratio (W/C) of 0.45 was selected for the investigation. Water cured concrete cubes of different ages were exposed to dilute sulphuric acid of 0.1% , 1% and 5 % concentrations. Amount of consumed or reacted acid by concrete with time, loss or gain of strength and weight with the amount of acid consumed per unit surface area were studied. Effect of continuous water curing and air drying inside the laboratory was also studied. After placing the concrete cubes in sulphuric acid solutions, the reduction in strength of sulphuric acid was measured at regular intervals using volumetric analysis. The procedure adopted for the estimation of strength of acid is explained. An algebraic equation is suggested for the consumption of acid with time. The constants of the equation were solved using the method of non-linear regression for the experimental observations. A procedure as to how to estimate the depth of penetration of acid, strength reduction factor, weight reduction factor,

quantity of cement required in one unit of concrete for a given exposure, depth of deterioration of concrete, and the coefficient of variation of strength is explained. In the process of estimation of the above quantities, the demand of sulphuric acid for saturated reaction with one kiloNewton of cement (V_R) is required. So the procedure for estimating V_R is explained in Appendix A. Applications of the results are discussed in Chapter IV with few examples.

3.2 Materials and Mix

The cement used in this investigation was analyzed to estimate the percentage of oxides present, using the procedure given by Vogel [103]. The chemical composition of the cement used in this investigation is presented in Table 3.1. Cement from the same lot was used throughout the investigation to avoid the variations in quality of cement. Locally available fine aggregate and granite base coarse aggregate were used conforming to IS 383 - 1970 [104]. The aggregates selected were assumed to be of non-reactive type. 100 x 100 x 100 mm concrete cubes were used in the investigation. The quantities of the mix, aggregate cement ratio 4.5 and water cement ratio 0.45, were weigh batched and thoroughly mixed manually. Cubes were compacted using table vibrator. They were demoulded after 24 hours.

and immediately put in curing tank for water curing.

TABLE 3.1 : CHEMICAL COMPOSITION OF CEMENT

Oxide	Weight %
SiO_2	27.24
CaO	53.30
Fe_2O_3	4.30
Al_2O_3	6.80
MgO	0.95
MnO	0.40
SO_3	0.13
Loss on Ignition	6.50

3.3 Experimentation

After proper water curing the cubes were exposed to dilute sulphuric acid of 0.1%, 1% and 5% concentrations. The strength of acid was measured at regular intervals and the depleted acid was replenished. For measuring the strength of acid volumetric analysis was used.

3.3.1 Procedure for Volumetric Analysis

To estimate the strength of sulphuric acid present in the sample, potassium hydroxide (KOH) was selected as a

base in the analysis [103]. Phenolphthalein was used as an indicator, which changes from colourless in acid medium to pink colour in alkaline medium. The strength of potassium hydroxide selected was approximately in the order of the strength of the sulphuric acid. The strength of potassium hydroxide was determined using the standard oxalic acid and phenolphthalein as an indicator. Let the normality of oxalic acid be N_1 . 5 cc of oxalic acid was taken into a conical flask and one drop of indicator was added. Potassium hydroxide was added to this solution drop by drop till the pink colour is just formed. Let potassium hydroxide consumed be V_b cc. Then

$$\begin{aligned} \text{Normality of KOH} = N_2 &= \frac{N_1 \times \text{Vol. of oxalic acid}}{V_b} \\ &= \frac{5 N_1}{V_b} \end{aligned} \quad (3.1)$$

Sample Calculation: Normality of oxalic acid is 0.1042 N and potassium hydroxide consumed by 5 cc of oxalic acid is 4.0 cc.

$$N_2 = \frac{5 N_1}{V_b} = \frac{5 \times 0.1042}{4.0} = 0.13025 \text{ N}$$

From the sample collected from the drums, 5 cc was taken into a conical flask. One drop of indicator was added to it. Then potassium hydroxide was added drop by drop till the pink colour just forms.

The acid number is the number of milliequivalents of KOH required to neutralize one kiloNewton of sample.

$$\text{Acid number} = \text{AN} = \frac{N_2 \times \text{atomic weight of KOH} \times \text{volume of KOH consumed}}{\text{volume of sample}}$$

$$= \frac{N_2 M_{\text{KOH}} V_b}{V_s} \quad (3.2)$$

$$\text{Moles of acid/cc} = c = \frac{\text{AN} \times 10^{-3}}{M_{\text{KOH}}} \quad (3.3)$$

where M_{KOH} - atomic weight of KOH, and

V_s - volume of sample.

3.3.2 Set-up

Polythene drums of capacity 90 litres each were used to store the cubes in a required concentration of sulphuric acid. There were four set-ups. In the first set-up concrete cubes water cured for 30 days were exposed to 0.1%, 1% and 5% concentration of sulphuric acid. In the second one concrete cubes water cured for 50 days were exposed to 0.1% and 1% concentration of sulphuric acid. In the third one, concrete cubes water cured for 90 days were exposed to 1% concentration, and in the last set-up 120 days water cured concrete cubes were exposed to 0.1% concentration. Cubes were placed staggeredly in the drums

to maximize the exposed surface. Along with each set-up equal number of cubes were kept for air drying inside the laboratory. Some cubes were left in the curing tank to study the effect of continuous water curing on strength and weight. Hereafter these cubes are referred as exposed to 0.0 % concentration.

3.3.3 Analysis of the Sample

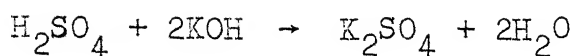
A sample of solution from each drum was collected at regular intervals to estimate the quantity of acid consumed by the concrete cubes. The solution was thoroughly stirred and collected into a sampling bottle. For each solution three samples were analyzed, and a mean value was taken for further computations. Necessary care and precautions were taken during the experiment. If c_0 is the initial strength of the solution, c_t is the strength at time t and V_0 is the initial amount of acid required for c_0 strength, then

$$\text{acid consumed} = \frac{c_0 - c_t}{c_0} V_0 \quad (3.4)$$

Sample Calculation: 0.4 litres of sulphuric acid was added to 39.6 litres of water. To neutralize 5 cc of this solution 13.7 cc of potassium hydroxide was consumed. Concrete cubes were exposed in this solution. After three days sample solution was collected again. This time 8 cc of potassium

hydroxide was required to neutralize 5 cc of the solution. Calculations for the amount of acid consumed by the concrete are:

$$AN_0 = \frac{N_2 M_{KOH} V_b}{V_s} = \frac{0.1303 \times 56.1 \times 13.7}{5} = 20.02$$



∴ each mole of H_2SO_4 needs 2 moles of KOH for neutralization.

moles of H_2SO_4 present in one litre of

$$\text{solution} = c_0 = \frac{20.02}{56.1 \times 2} = 0.1784 \text{ moles of } H_2SO_4/\text{litre.}$$

where the subscript to AN and c refers to the age in days at which the sample is analyzed.

$$AN_3 = \frac{0.1303 \times 56.1 \times 8}{5} = 11.70$$

$$c_3 = \frac{11.70}{56.1 \times 2} = 0.10424 \text{ moles of } H_2SO_4/\text{litre.}$$

Acid consumed in 3 days

$$= \frac{c_0 - c_3}{c_0} V_0 = \frac{0.1784 - 0.10424}{0.1784} \times 0.4 = 0.166 \text{ l.}$$

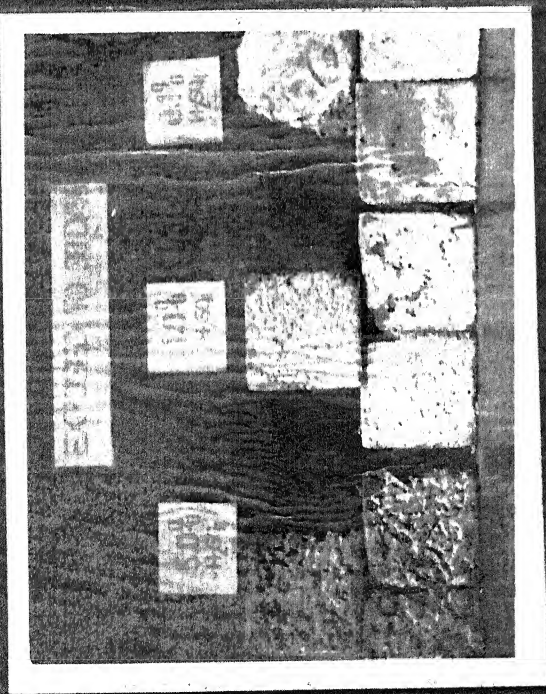
Therefore, approximately 0.17 litres of acid was added to the solution and thoroughly mixed to get uniform solution.

Similar procedure was employed for all the samples and at all ages. At specified intervals the strength, weight and

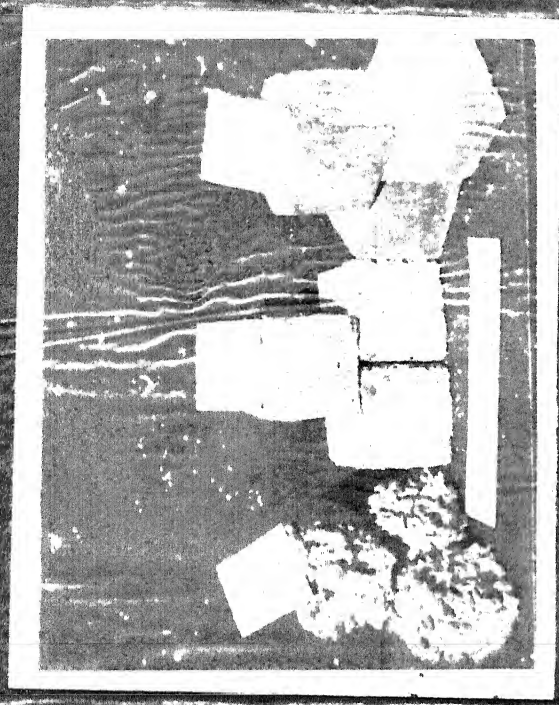
depth of concrete dissolved were measured. In 1% and 5% concentrations of sulphuric acid, the surface of the cubes was irregular, because it was dissolved (Figs. 3.1 - 3.4). In such cases cube faces were capped with neat cement paste as specified by IS 516 -1959 [105] to make the contact surface plain before testing for strength.

3.4 Results and Discussion

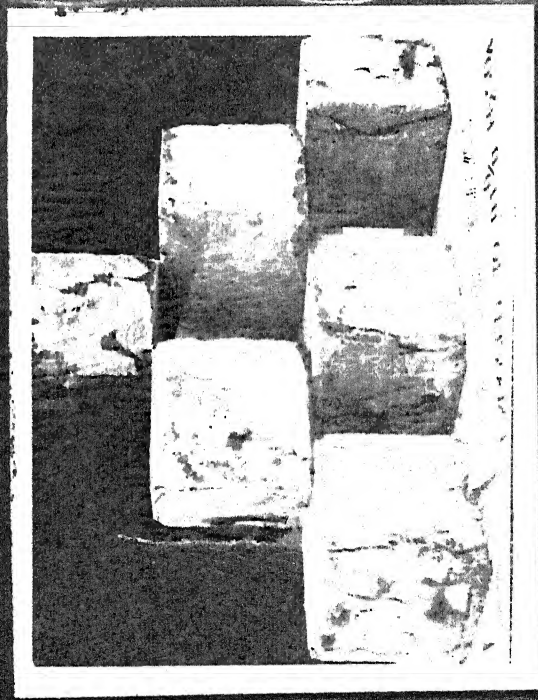
While titrating for determination of the strength of acid, yellow colour appeared prior to the pink colour, which indicates the formation of sulphates during the sulphuric acid attack. The sulphate formation on the surface (needle shaped fins) of concrete exposed to 0.1% concentration of sulphuric acid for 250 days can be seen in Fig. 3.4b. In the case of concrete cubes exposed to 1% and 5% concentrations of sulphuric acid, the dissolving attack was so dominant, the surface gets dissolved fast and hence no fins on the surface were observed (Figs. 3.1 - 3.4). Yet the pink colour formation during titration was commonly observed. Figures 3.1 - 3.4 show the concrete cubes exposed to different ages under 0.1%, 1% and 5% concentrations. In case of concrete exposed to 0.1% concentration of sulphuric acid no visual changes could be found. However surface became whitish and the needle like fins were observed on the surface after 60 days of exposure. In case of concrete



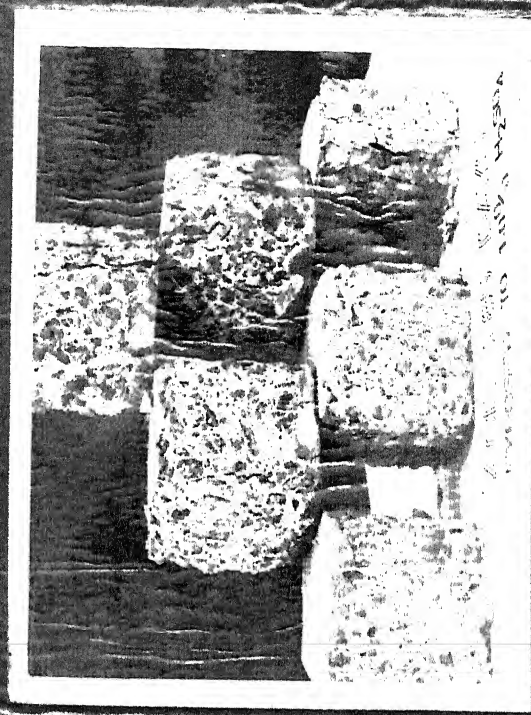
(a)



(b)



(c)



(d)

FIG. 3.1 : CONCRETE CUBES EXPOSED TO SULPHURIC ACID

(a) 0.1%, 1% AND 5% FOR 30 DAYS (b) 0.1%, 1%, 5%

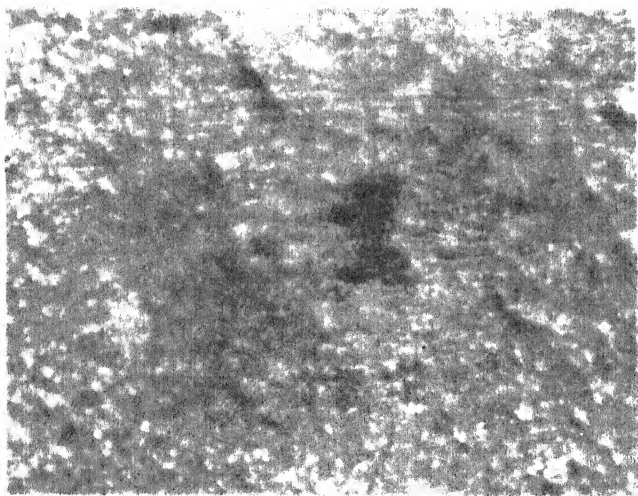
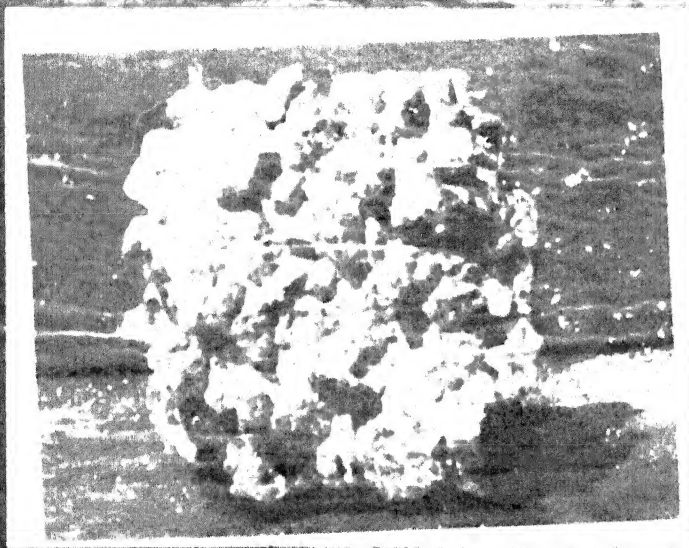
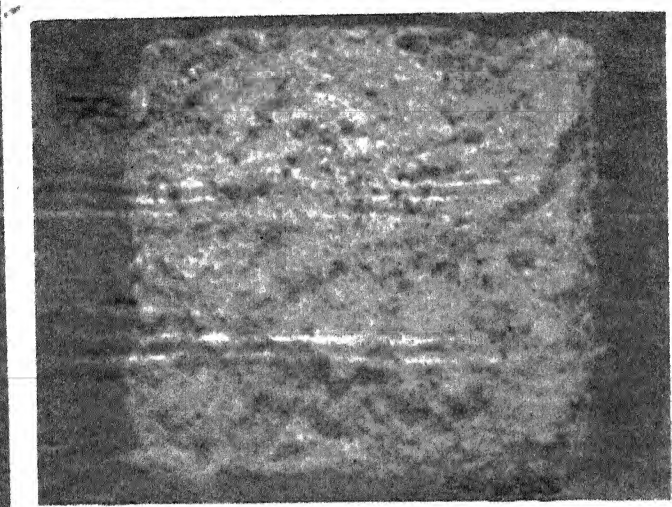


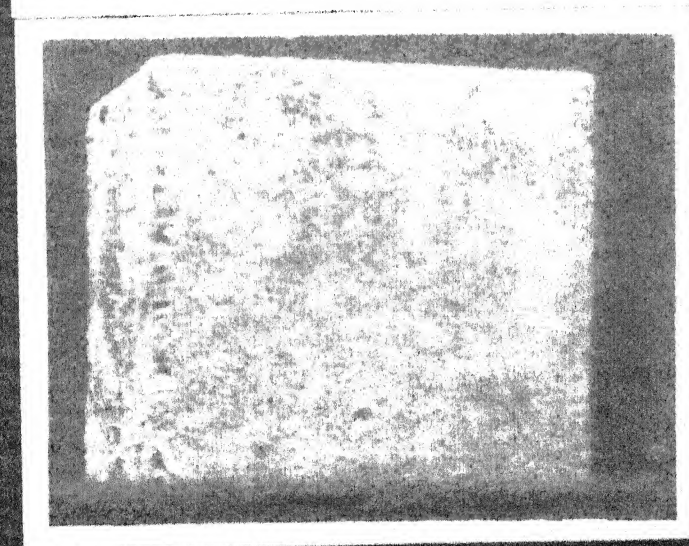
Fig. 3.2 : SURFACE OF CONCRETE CUBES EXPOSED TO SULPHURIC ACID FOR 30 DAYS (a) 5% CONCENTRATION (b) 1% CONCENTRATION (c) 0.1% CONCENTRATION



(a)



(b)



(c)

FIG. 3.3: SURFACE OF CONCRETE CUBES EXPOSED TO SULFURIC ACID FOR 85 DAYS (a) 5% CONCENTRATION (b) 1% CONCENTRATION (c) 0.1% CONCENTRATION

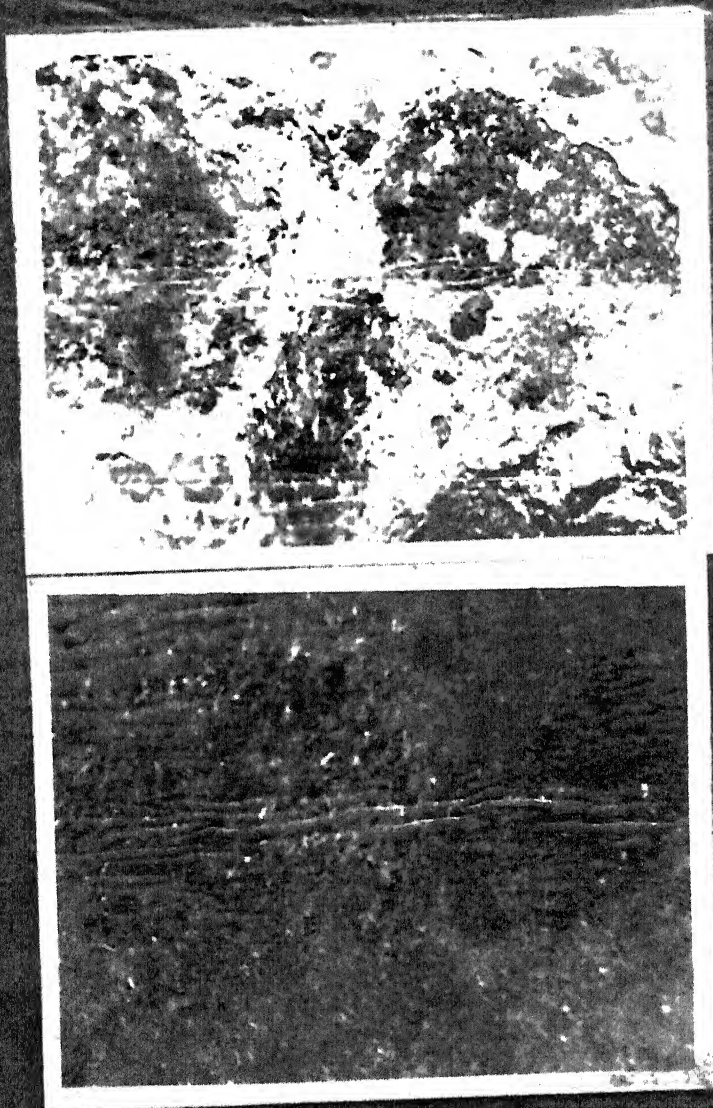


FIG. 3.4 : SURFACE OF CONCRETE CUBES EXPOSED TO SULPHURIC
ACID FOR 295 DAYS (a) 1% CONCENTRATION
(b) 0.1% CONCENTRATION

exposed to 1% concentration of sulphuric acid, surface started to soften in 30 days. Initiation of surface scaling was observed at the age of 60 days. In 95 days the total surface mortar became soft. About 95 % of the surface mortar was scalled off and aggregate was fully exposed when observed on 180th day. In further observations, dissolving of mortar deep inside leaving the projection of particles was observed. In 5% exposure coarse aggregate /the effect is so rapid, the surface mortar was dissolved within very short time. When observed on 30th day, surface mortar was dissolved fully and the coarse aggregate particles projected out. During further observations, the mortar was found to be dissolving further inside and part of the coarse aggregate on the surface was lost due to break of bond (Fig.3.1b).

3.4.1 Consumption of Acid Per Unit Surface Area with Time

The consumption of acid per unit surface area of exposed concrete can be expressed as:

$$V' = A [1 - \exp (- Bt^C)] \quad (3.5)$$

where V' - acid consumed by a unit surface area of concrete
 t - time, and

A,B,and C- regression constants.

From Eq.(3.5) , V' is zero for t equal to zero and for a very

large value of t , V' approximately equals to A . Therefore the value of A is calculated approximately, assuming that the member fails if the concrete upto a depth 100 mm is fully reacted. In the calculation of A , the type of acid is essential rather than the concentration of acid for which the concrete is exposed. Whereas B and C are the constants, which depend upon the concentration of acid and the properties of the concrete. The quality and quantity of cement are also included in the calculation of A . The quantity of cement present in one square meter of surface area and 100 mm depth of concrete is $0.1 q$ kN, where q is the cement content in kiloNewton per cubic meter of concrete. Then the quantity of sulphuric acid (A , l/m^2) required for saturated reaction with $0.1 q$ kN is :

$$A = 0.1 q V_R \quad l/m^2 \quad (3.6)$$

where V_R is acid required for saturated reaction with one kiloNewton of cement. The procedure for computing V_R is given in Appendix-A.

$$\therefore A = 0.1 \times 4.26 \times 73.4 = 31.2 \approx 30 \quad l/m^2$$

However, the selection of 100 mm depth is arbitrary, so for different values of A , B and C were determined. If V is the actual amount of acid consumed by concrete per unit surface area, the error at any point (η) is :

$$\eta = V_i - V'_i$$

The cumulative least square error is (η_c^2) :

$$\eta_c^2 = \sum_i (V_i - V'_i)^2 = \sum_i [V_i - A + A \exp(-Bt_i^C)]^2 \quad (3.7)$$

where t_i - i th time in years, and

V'_i - acid consumed upto t_i th year.

Equation (3.7) is differentiated with respect to B and C and independently equated to zero. The two equations obtained are solved for B and C, which fits with a minimum least square error [106] .

$$\frac{\partial \eta_c^2}{\partial B} = \sum_i [V_i - A + A \exp(-Bt_i^C)] \exp(-Bt_i^C) t_i^C = 0 \quad (3.8)$$

$$\frac{\partial \eta_c^2}{\partial C} = \sum_i [V_i - A + A \exp(-Bt_i^C)] \exp(-Bt_i^C) t_i^C \ln t_i = 0 \quad (3.9)$$

Simplification of the above two equations gives:

$$\begin{aligned} A \left[\sum_i t_i^C \exp(-Bt_i^C) - \sum_i t_i^C \exp(-2Bt_i^C) \right] \\ - \sum_i V_i \exp(-Bt_i^C) t_i^C = 0 \end{aligned} \quad (3.10)$$

$$\begin{aligned} A \left[\sum_i t_i^C \exp(-Bt_i^C) \ln t_i - \sum_i t_i^C \exp(-2Bt_i^C) \ln t_i \right] \\ - \sum_i V_i t_i^C \ln t_i \exp(-Bt_i^C) = 0 \end{aligned} \quad (3.11)$$

The above two non-linear equations (Eqs. (3.10) and (3.11)) are of the form $F_1(B, C) = 0$ and $F_2(B, C) = 0$. These were solved using iterative technique. A flow chart for the

Procedure adopted to solve the above two equations is given in Fig. 3.5. Wherever necessary, Regula falsi [107] method was used to solve a non-linear equation in single variable .

If C is taken as 0.5 , the above two equations reduce to a single non-linear equation with B as an unknown.

$$A \left[\sum_i T_i \exp(-BT_i) - \sum_i T_i \exp(-2BT_i) \right] - \sum_i V_i T_i \exp(-BT_i) = 0 \quad (3.12)$$

where $T_i = t_i^{0.5}$

The above equation was solved using Regula falsi method to get B . The constants A, B and C and the normalised cumulative least square error (η_c) for each concentration are tabulated in Table 3.2 (page 81). Using Figs. 3.3, 3.8 one can interpolate B and C for a given value of A. The range of A is 5 to 40. The rate of change of B or C for the value of A beyond 30 is very small.

In Figs. 3.9 - 3.11 , the experimental values of acid consumed (l/m^2) with time (weeks) and the corresponding least square fit values are presented. The percentage error at each point ranges from 0 to 9% for the concrete exposed to 0.1 % concentration of sulphuric acid. But the error is higher in first two weeks in case of concrete cured for 30 days, first four weeks in 50 days water cured concrete

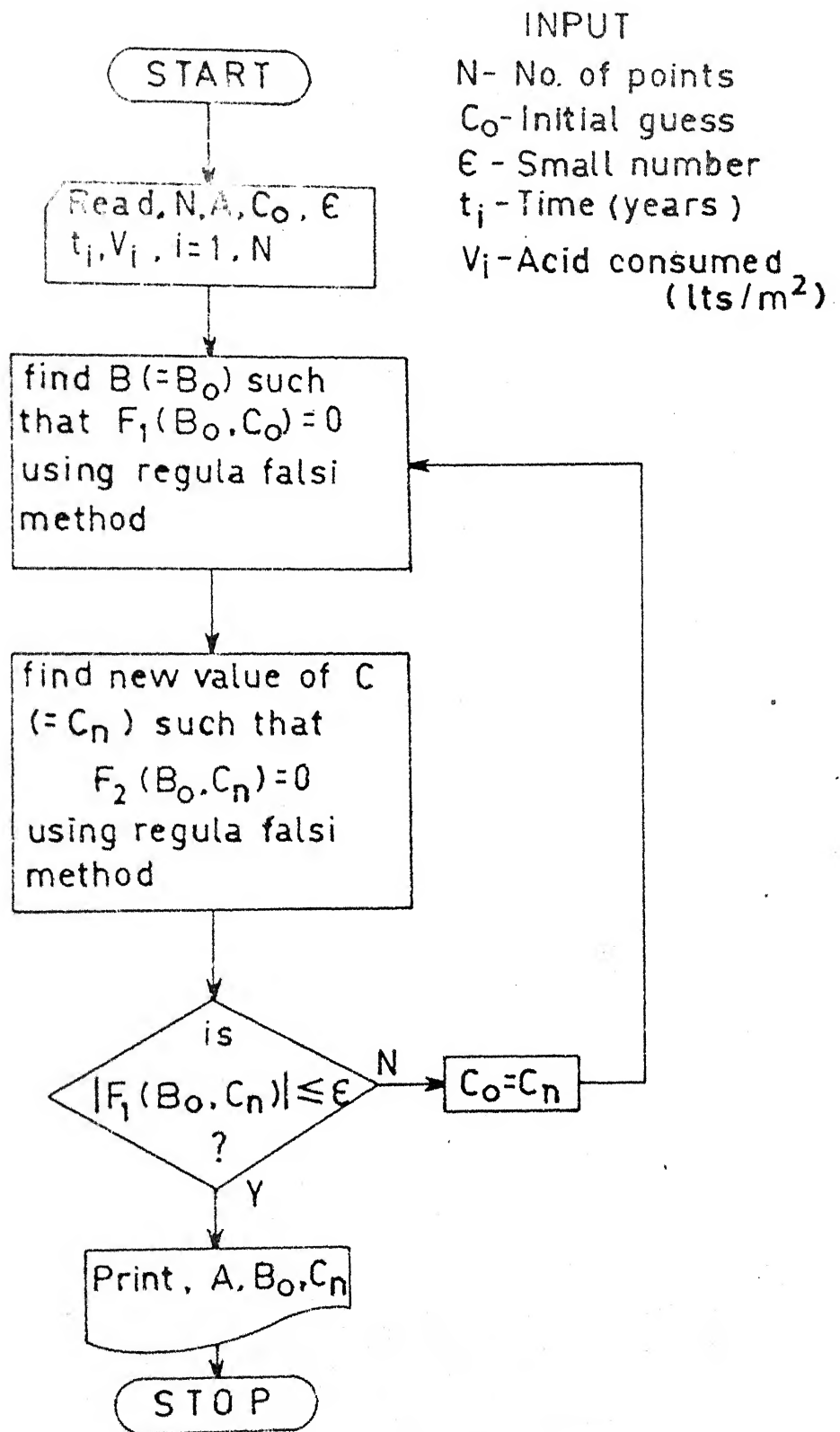


FIG.3.5 FLOW CHART FOR SOLVING B AND C

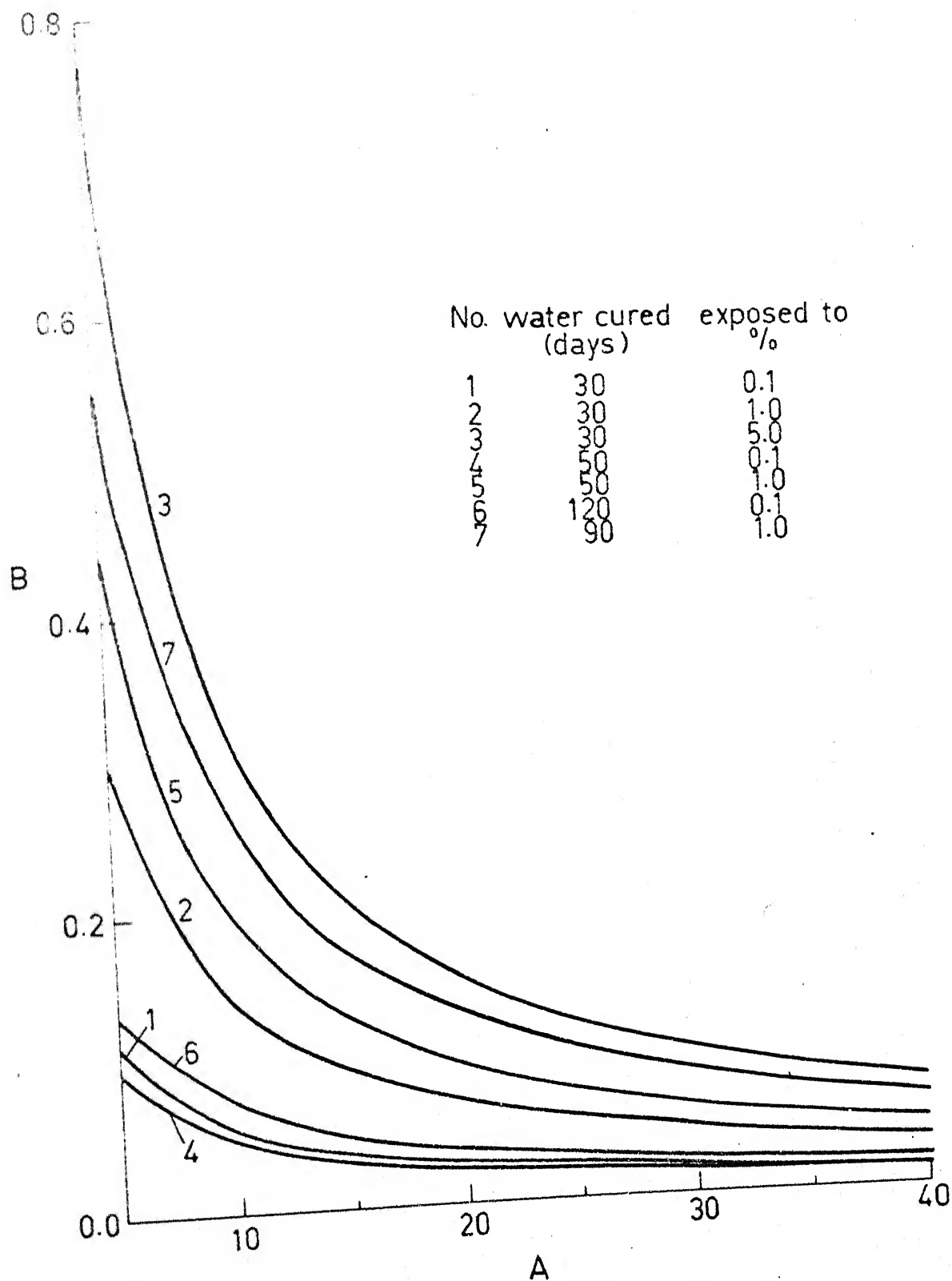


FIG. 3.6 VARIATION OF B WITH A

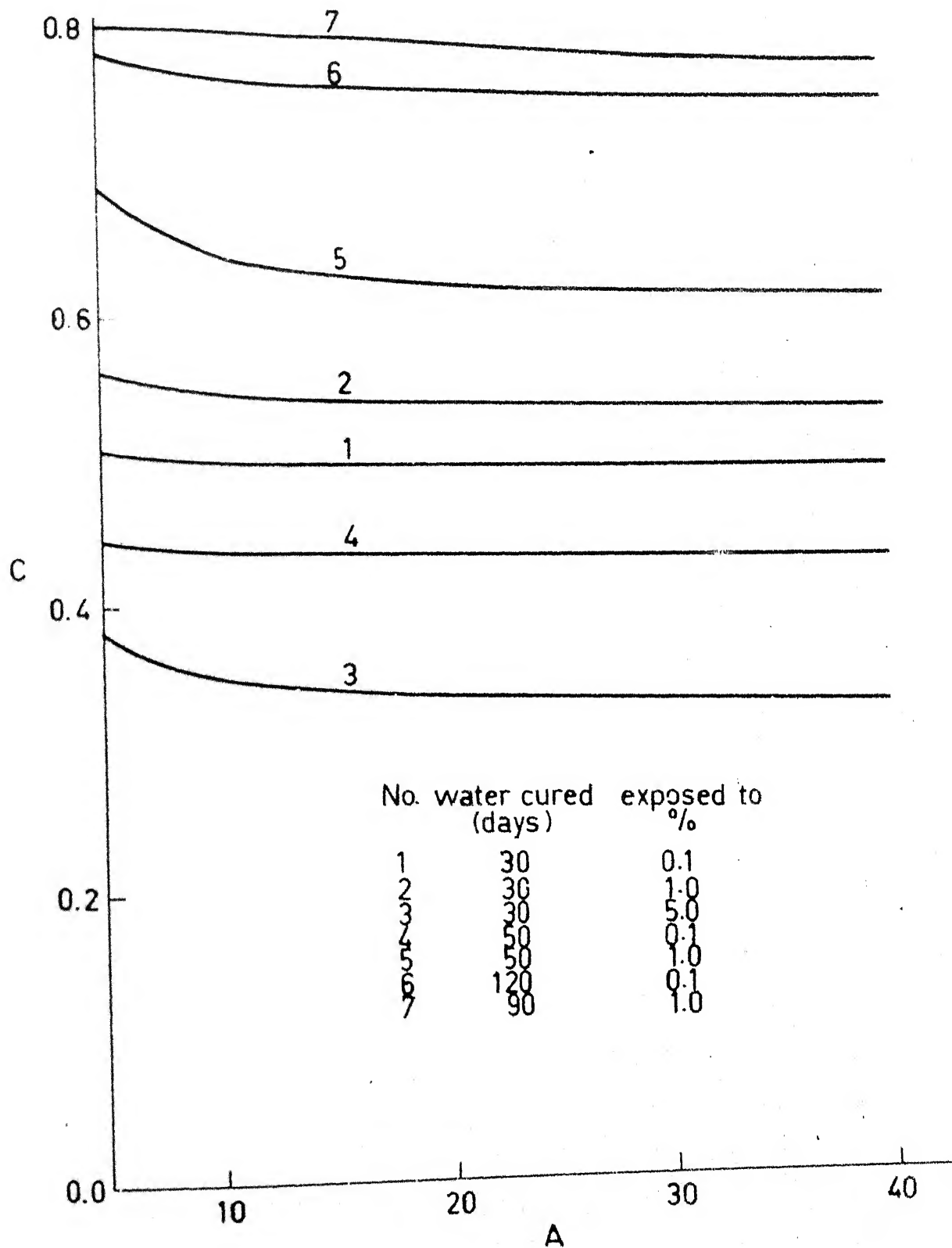


FIG. 3.7 VARIATION OF C WITH A

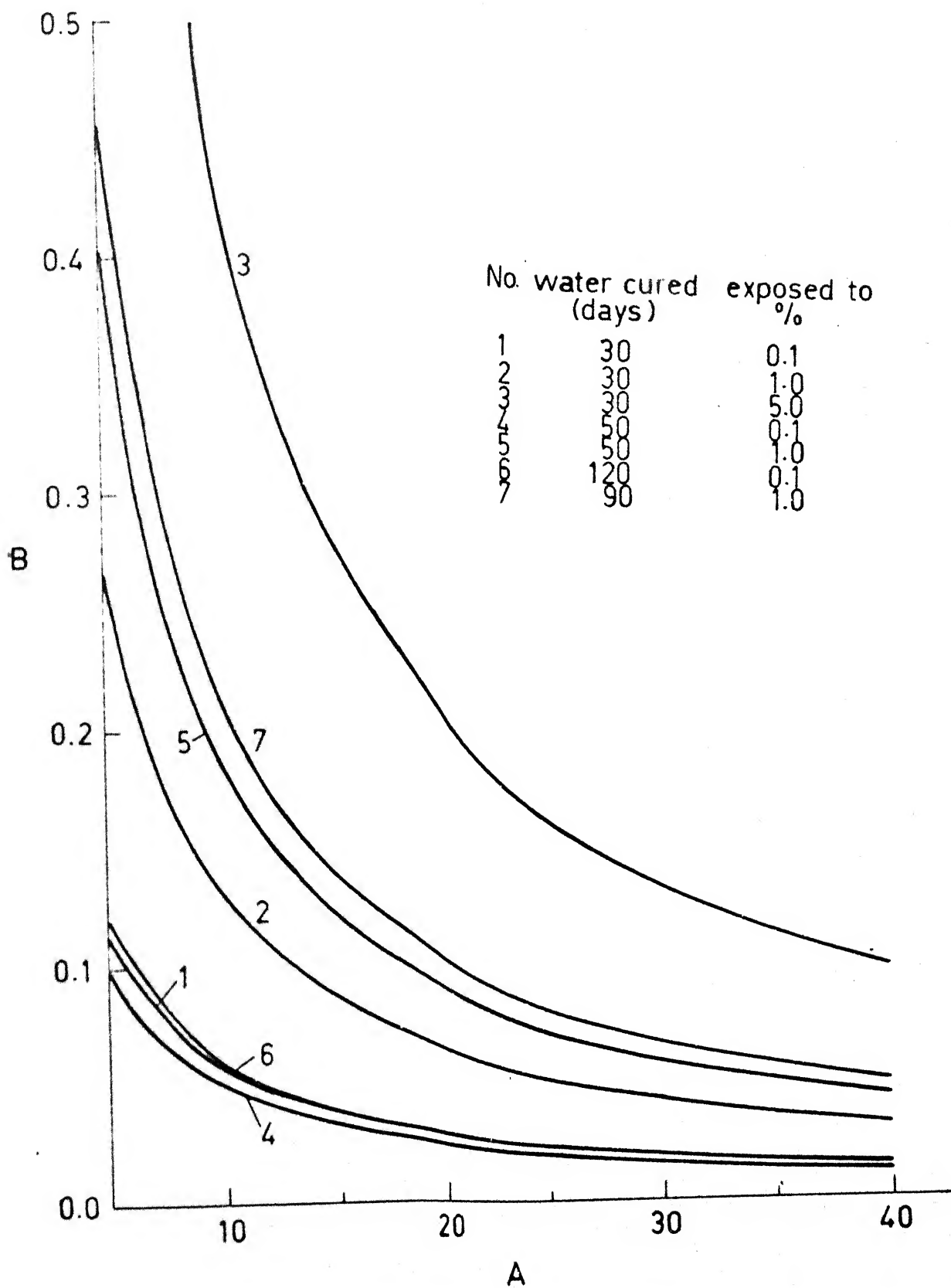


FIG. 3.8 VARIATION OF B WITH A FOR $C=0.5$

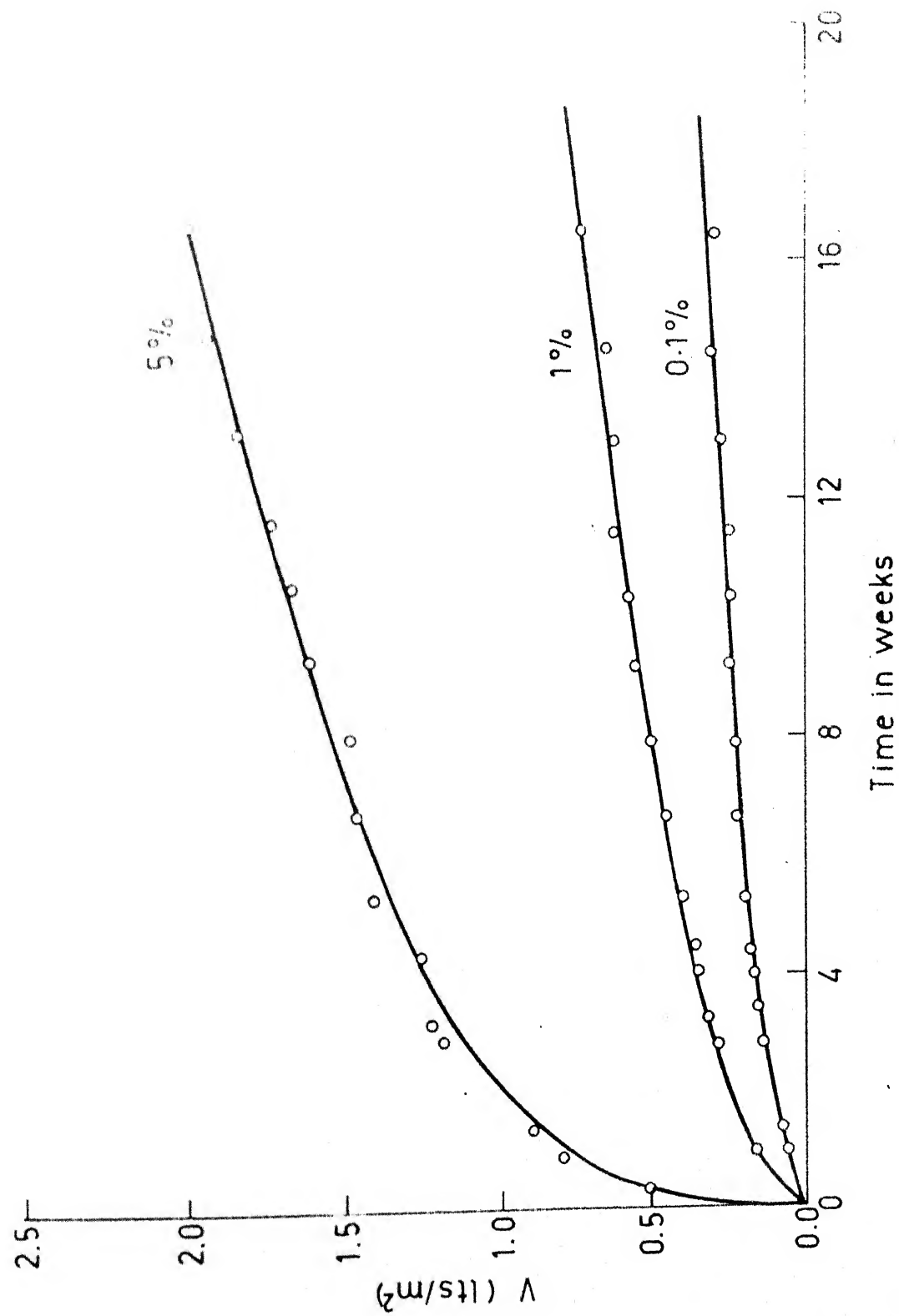


FIG 3.9 VARIATION OF CONSUMPTION OF ACID WITH TIME (30 DAYS WATER CURED CONCRETE)

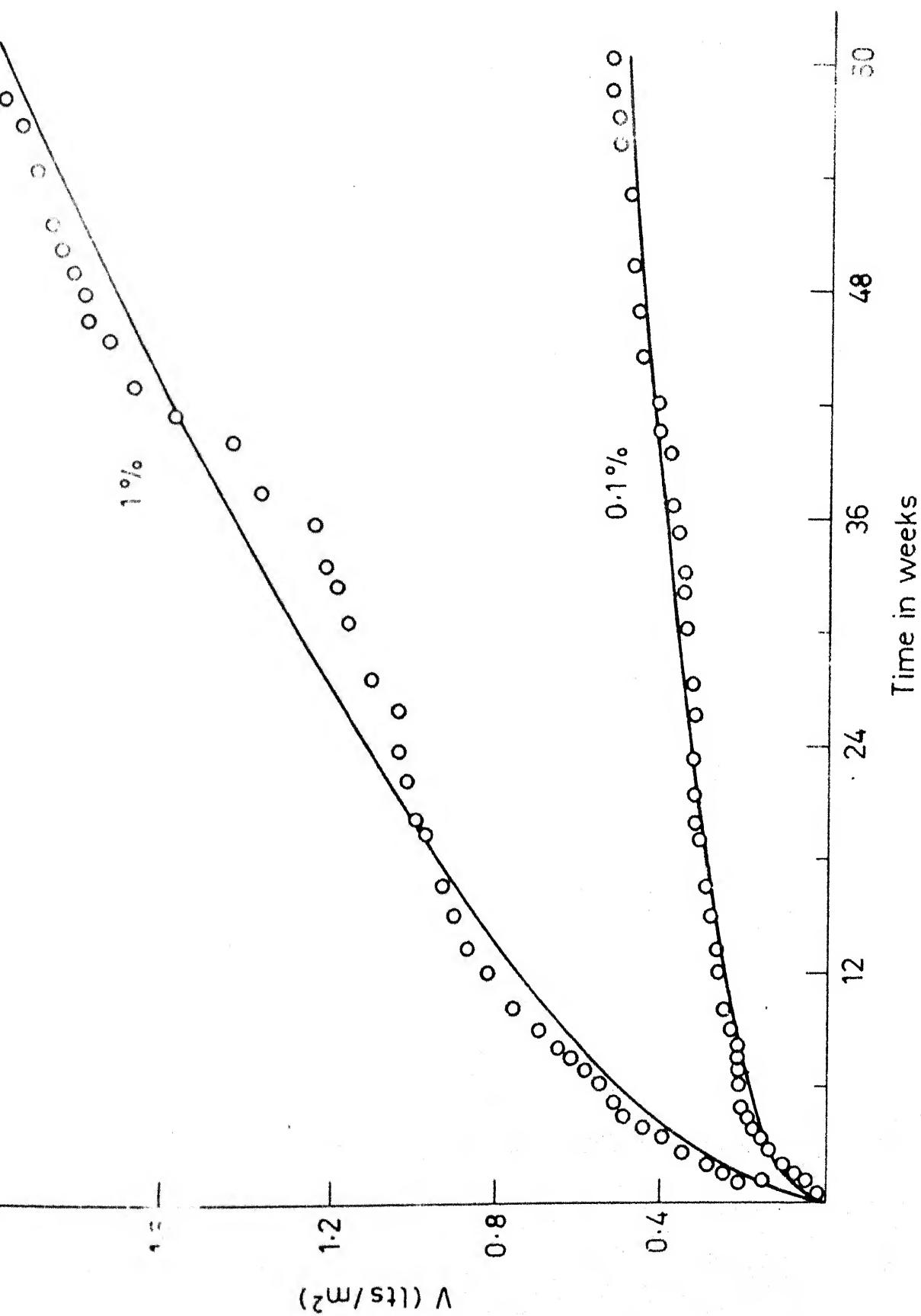


FIG. 3.10 VARIATION OF CONSUMPTION OF ACID WITH TIME (50 DAYS WATER CURED CONCRETE)

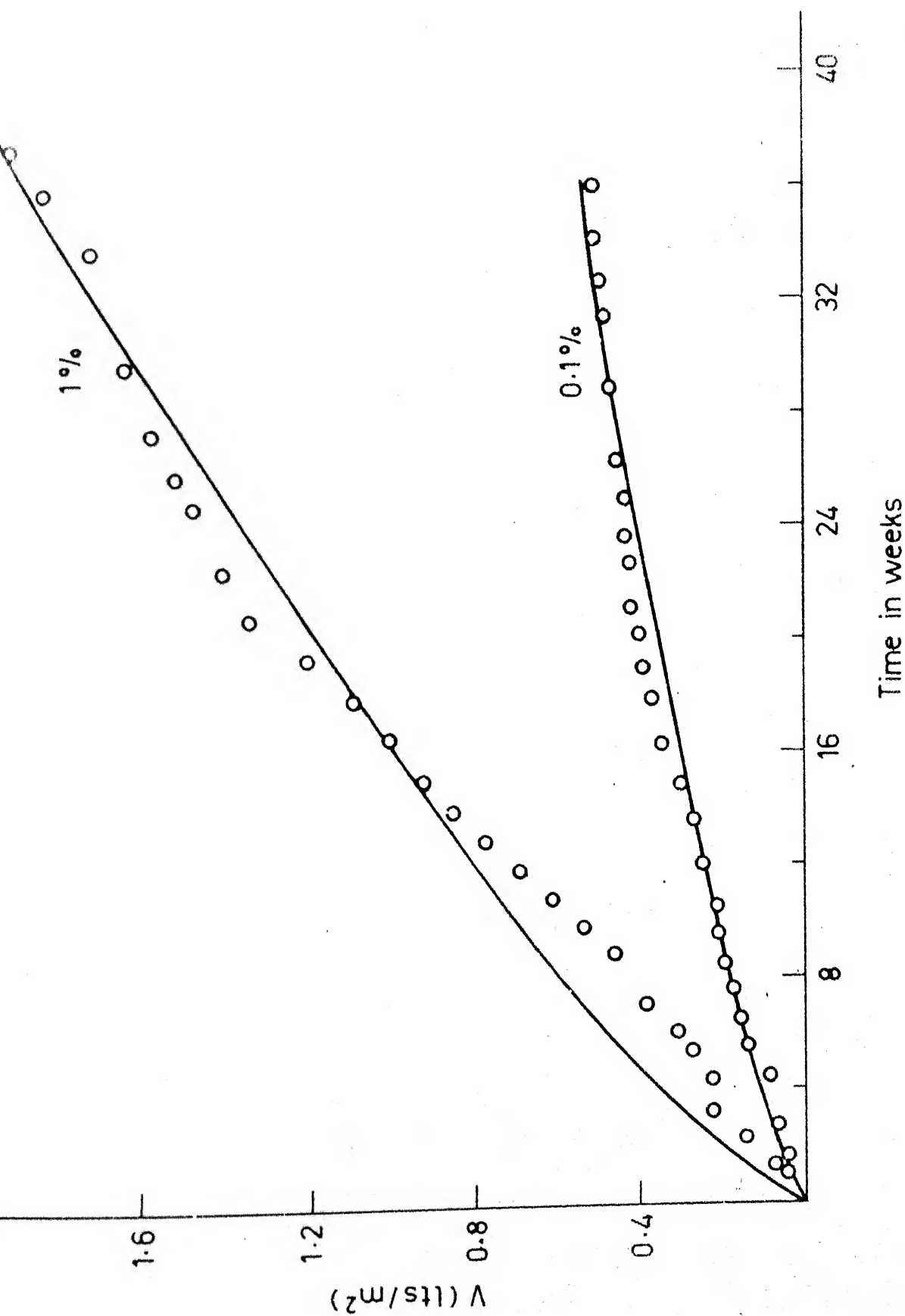


FIG. 3.11 VARIATION OF CONSUMPTION OF ACID WITH TIME (90/120 DAYS WATER CURED CONCRETE)

and first seven weeks in 120 days water cured concrete. For the concrete exposed to 1% concentration, the range is 0 to 4.5 % for 30 days water cured concrete. It is higher in first three weeks and ten weeks for 50 days and 90 days water cured concretes respectively. Beyond the above specified period, the range is 0 to 11% . The percentage error ranges from 0 to 4.5% at all points in case of 30 days water cured concrete exposed to 5% concentration. However, except in 90 days water cured concrete exposed to 1% and 120 days water cured concrete exposed to 0.1% concentrations the maximum error is only 30% . Figure 3.12 is a combined plot of all the least square fit values showing the variation of consumption of acid with time. In case of 0.1% exposure, the advantage of water curing for higher period is nominal.

Figure 3.13 shows the variation of values of B and C with concentration of sulphuric acid for 30 days water cured concrete. It can be used to obtain the values of B and C for a given concentration. Upto and including 1% the value of C is taken as 0.5 . The value of A is taken as 30 for all concentrations.

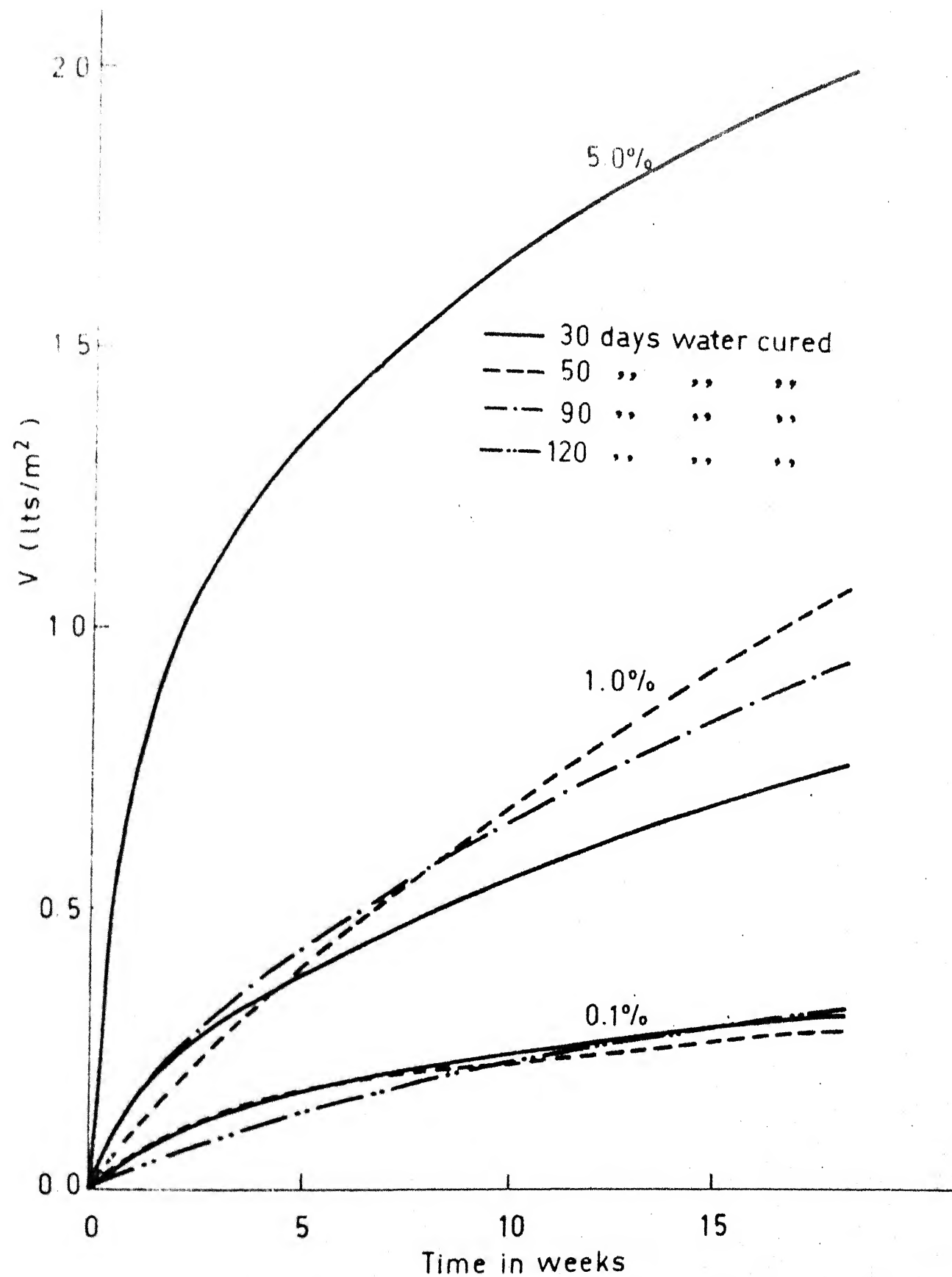


FIG. 3-12 VARIATION OF CONSUMPTION OF ACID WITH TIME

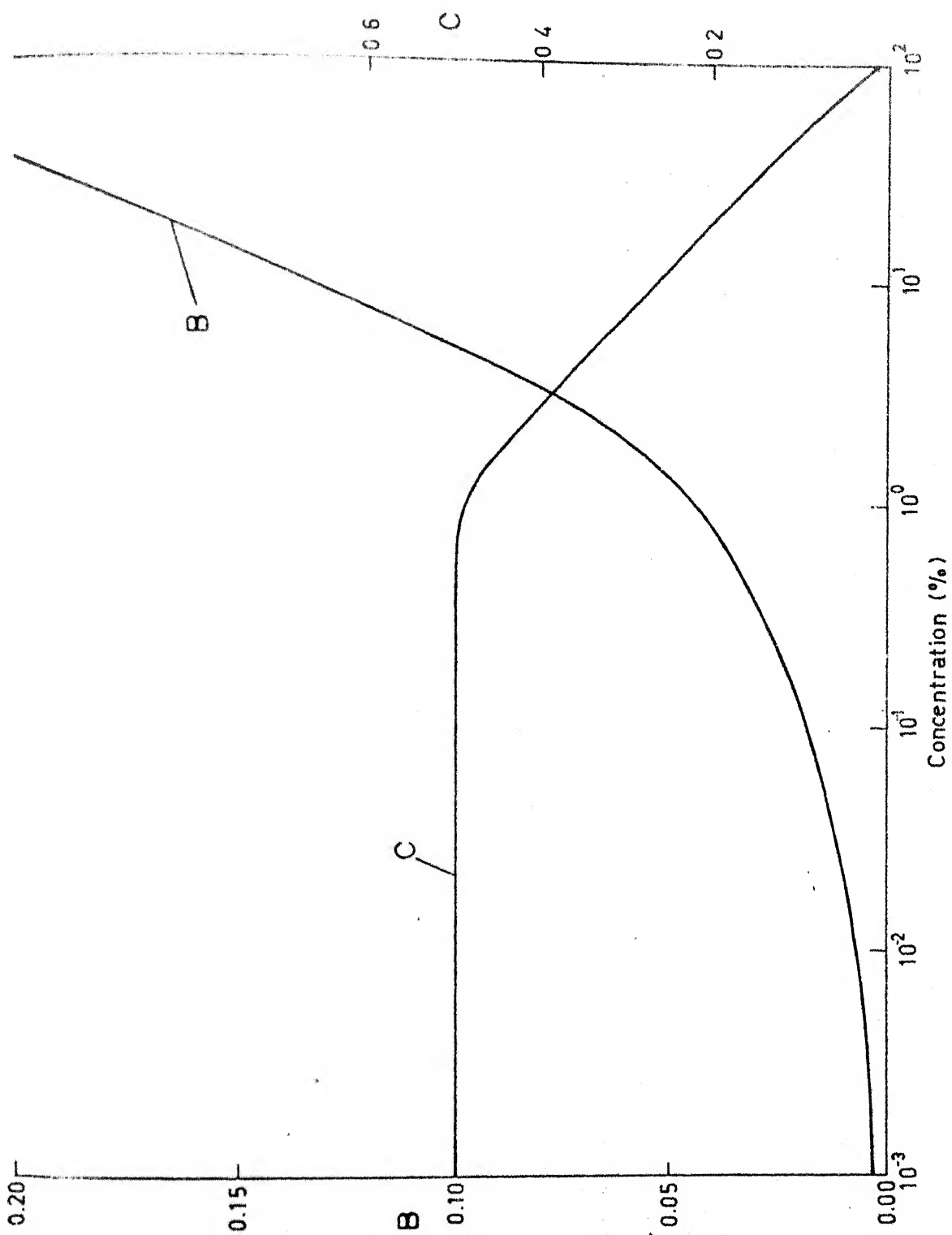


FIG. 3.13 VARIATION OF B AND C WITH CONCENTRATION OF ACID.

TABLE 3.2 : VALUES OF A, B, C AND η_c^2

Water curing in days	Concentration %	A = 30.0			A = 30.0; C = 0.5	
		B	C	η_c^2	B	η_c^2
30	0.1	0.01875	0.4984	1.6660	0.01832	1.699
	1.0	0.04604	0.5400	0.0066	0.04293	0.0624
	5.0	0.10042	0.3338	0.0324	0.13530	0.396
50	0.1	0.01576	0.43412	2.9150	0.01624	1.433
	1.0	0.06250	0.61987	0.5160	0.05938	0.944
90	1.0	0.03562	0.78627	3.3960	0.06775	30.671
120	0.1	0.02451	0.75809	1.5130	0.01944	17.529

3.4.2 Strength of Exposed Concrete

Strength of concrete, water cured for 30 days and exposed to different concentrations of sulphuric acid, with time is shown in Fig. 3.14. The strength at all ages was normalized with 28 days concrete strength. In the same plot, strength of water cured and air dried concretes are also shown. There has been an increase in strength of

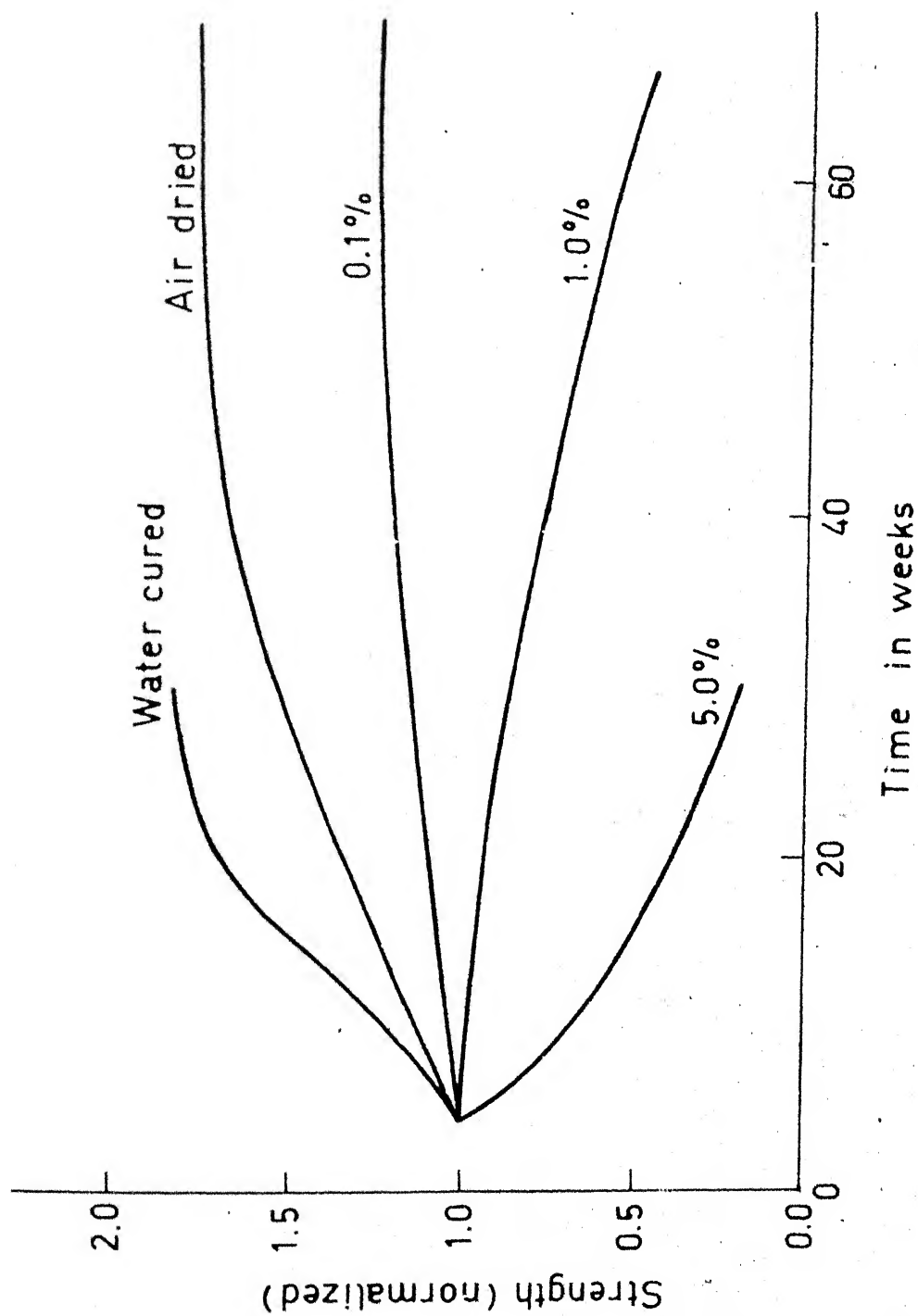


FIG. 3.14 VARIATION OF STRENGTH OF CONCRETE WITH TIME

concrete in case of air dried or water cured concrete. An increase in strength was also observed in concrete exposed to 0.1% concentration. However its strength compared to that of air dried or water cured one is on the lower side. Considerable loss of strength was observed in case of concrete exposed to 1 % and 5% concentrations. If the strength of concrete exposed to acid was compared with the strength of air dried or water cured concrete, consistent deterioration can be observed irrespective of the concentration of acid (Table 3.3).

TABLE 3.3 : CUBE STRENGTH OF ACID EXPOSED CONCRETE w.r.t.
STRENGTH OF AIR DRIED CONCRETE

Concentration of acid (%)	Age in weeks*				
	4	12	24	36	52
0.0	1.050	1.224	1.217	-	-
0.1	0.921	0.863	0.767	0.724	0.579
1.0	0.888	0.756	0.592	0.463	0.343
5.0	0.674	0.607	0.167	-	-

* Number of weeks exposed to acid after 30 days of water curing

The loss or gain of strength of concrete with the amount of acid consumed is shown in Fig. 3.15. A second order polynomial is fitted for this curve.

$$r_f = \frac{f_{ce}}{f_{co}} = C_1 + C_2 V + C_3 V^2 \quad (3.13)$$

where f_{ce} - strength of exposed concrete

f_{co} - 28 days strength of concrete

r_f - strength reduction factor

C_1, C_2 and C_3 - polynomial constants

The error at any point (η_f) is

$$\eta_f = (r'_{fi} - C_1 - C_2 V_i - C_3 V_i^2)$$

where r'_{fi} is the strength reduction factor at i th time,

obtained experimentally. The cumulative least square error

$$\eta_{fc}^2 = \sum_i (r'_{fi} - C_1 - C_2 V_i - C_3 V_i^2)^2 \quad (3.14)$$

Equation (3.14) is differentiated w.r.t. C_1 , C_2 and C_3 are independently equated to zero:

$$\frac{\partial \eta_{fc}^2}{\partial C_1} = \sum_i (r'_{fi} - C_1 - C_2 V_i - C_3 V_i^2) = 0 \quad (3.15)$$

$$\frac{\partial \eta_{fc}^2}{\partial C_2} = \sum_i (r'_{fi} - C_1 - C_2 V_i - C_3 V_i^2) V_i = 0 \quad (3.16)$$

$$\frac{\partial \eta_{fc}^2}{\partial C_3} = \sum_i (r'_{fi} - C_1 - C_2 V_i - C_3 V_i^2) V_i^2 = 0 \quad (3.17)$$

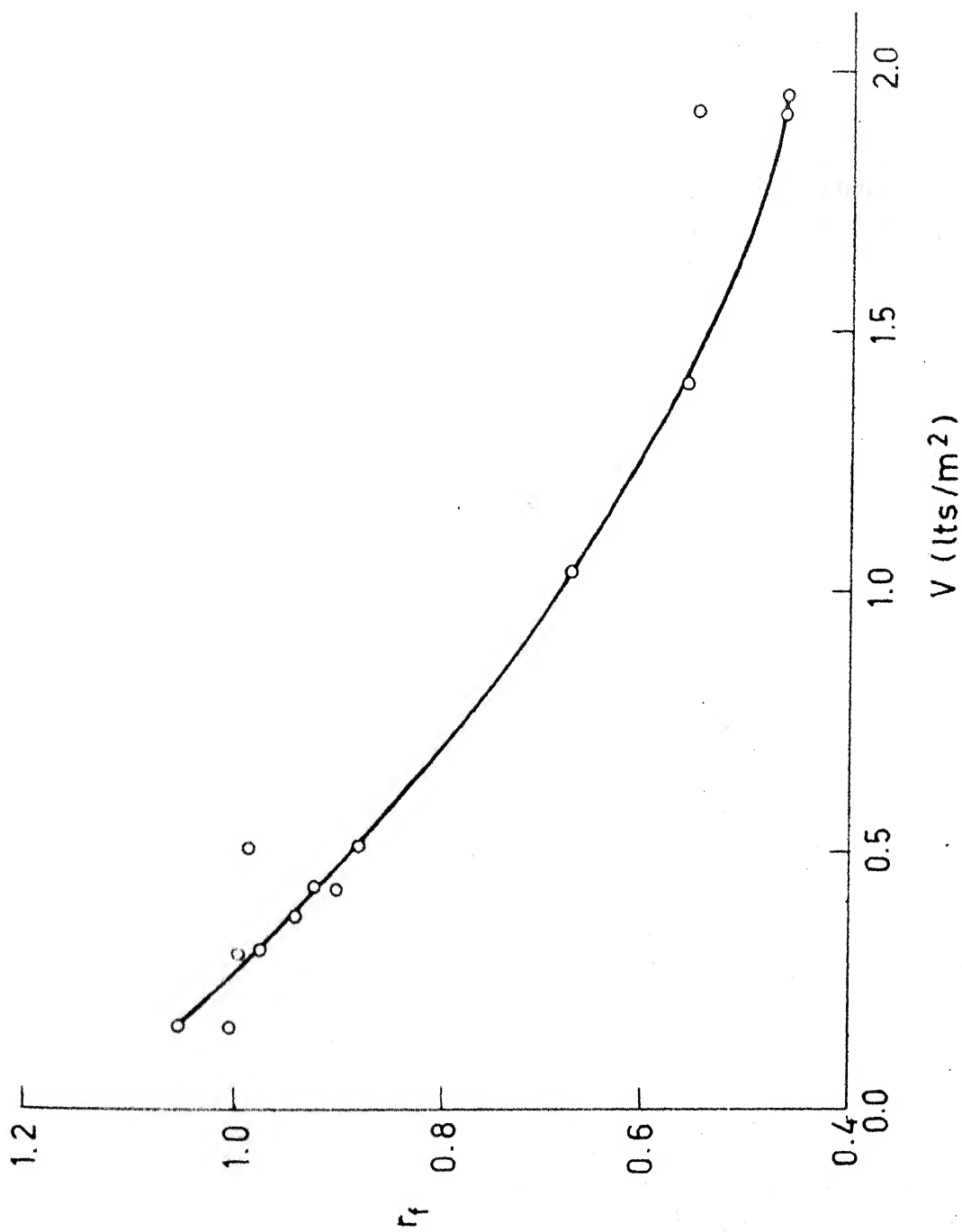


FIG. 3.15 VARIATION OF STRENGTH OF CONCRETE WITH
ACID CONSUMED

Simplification of Eq. (3.15) - Eq. (3.17) gives :

$$\sum_i C_1 + C_2 \sum_i V_i + C_3 \sum_i V_i^2 = \sum_i r'_{fi} \quad (3.18)$$

$$C_1 \sum_i V_i + C_2 \sum_i V_i^2 + C_3 \sum_i V_i^3 = \sum_i r'_{fi} V_i \quad (3.19)$$

$$C_1 \sum_i V_i^2 + C_2 \sum_i V_i^3 + C_3 \sum_i V_i^4 = \sum_i r'_{fi} V_i^2 \quad (3.20)$$

Equations (3.18) - (3.20) are solved for C_1 , C_2 and C_3 .

$$\therefore r_f = 1.14 - 0.5741 V + 0.1163 V^2 \quad (3.21)$$

The normalized cumulative least square error in this polynomial fit is 0.244. In addition to this, third order polynomial and exponentially decaying type curves are also tried. The equations in third order polynomial (Eq. (3.22)) and exponential fit (Eq. (3.23)) are :

$$r_f = 0.9933 + 3.389 V - 116.0 V^2 + 430 V^3 \quad (3.22)$$

$$r_f = \exp(-0.353 V^{1.222}) \quad (3.23)$$

The normalized cumulative least square error in third order polynomial fit is 0.10. Whereas it is 0.321 in exponential type fit. Equation 3.21 is used in further applications.

3.4.3 Weight of Exposed Concrete

Figure 3.16 shows the variation of weight of the exposed concrete (water cured for 30 days and then exposed to different concentrations) with time. The weight of concrete at all ages was normalized with 28 days weight of concrete. The increase in weight, 3.5 % in 0.1% exposed concrete, could be due to the dominant sulphate attack. Only a marginal change in weight of water cured concrete was observed. A loss of weight of about 2% in air dried concrete was observed, which could be due to the evaporation of moisture from concrete due to continuous air drying. Figure 3.17 shows the loss or gain of weight with the amount of acid consumed by concrete. The weight of concrete is normalized with the corresponding air dried concrete, (tabulated in Table 3.4).

TABLE 3.4 : WEIGHT OF ACID EXPOSED CONCRETE w.r.t. THAT
OF AIR DRIED CONCRETE

Concen- tration (%)	Age in weeks*				
	4	12	24	36	52
0.0	1.000	1.005	1.025	-	-
0.1	1.015	1.025	1.046	1.061	1.062
1.0	1.020	0.985	0.927	0.867	0.773
5.0	0.865	0.690	0.538	-	-

* Number of weeks exposed to acid after 30 days of water curing.

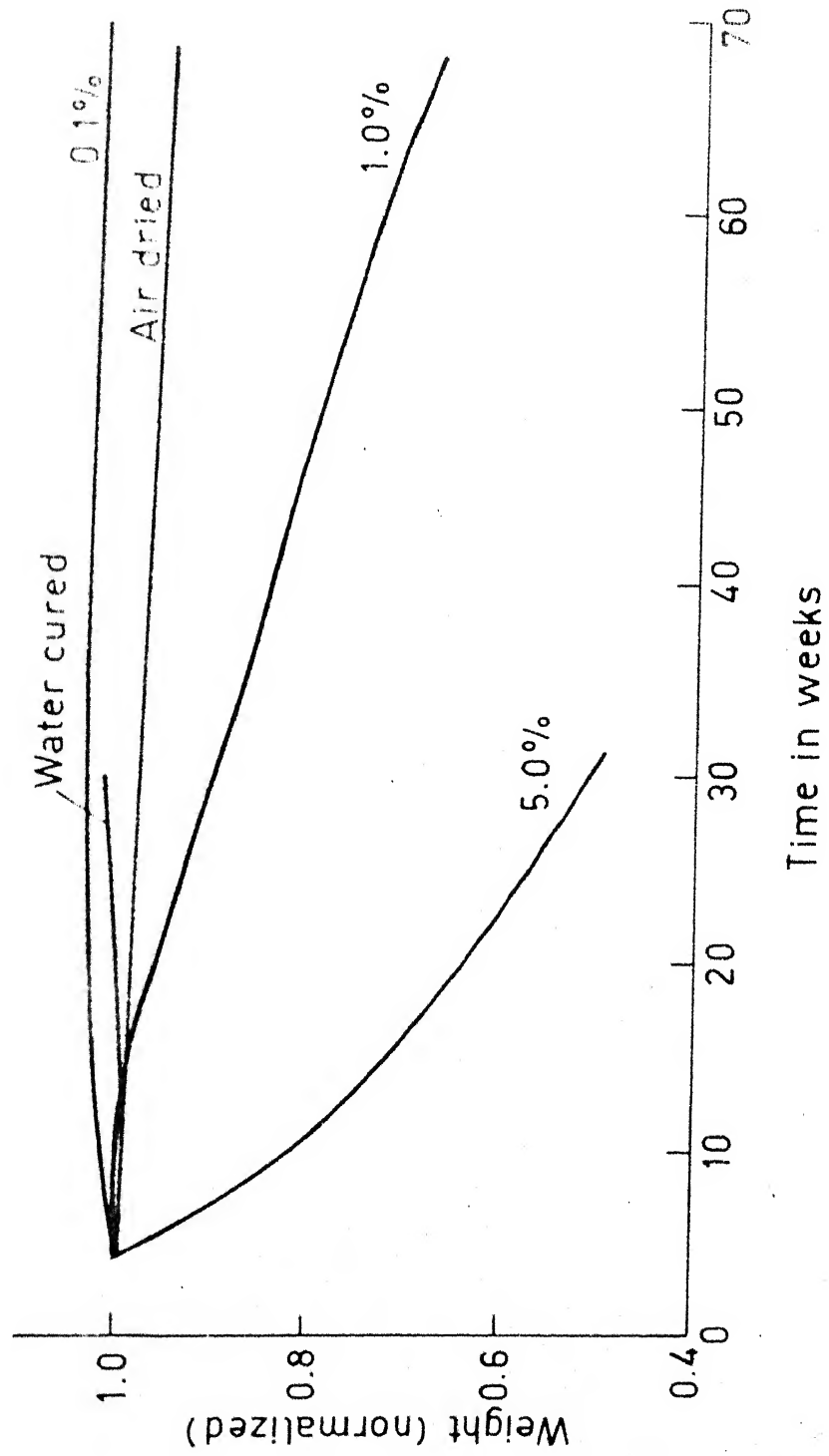


FIG. 3.16 VARIATION OF WEIGHT OF CONCRETE WITH TIME

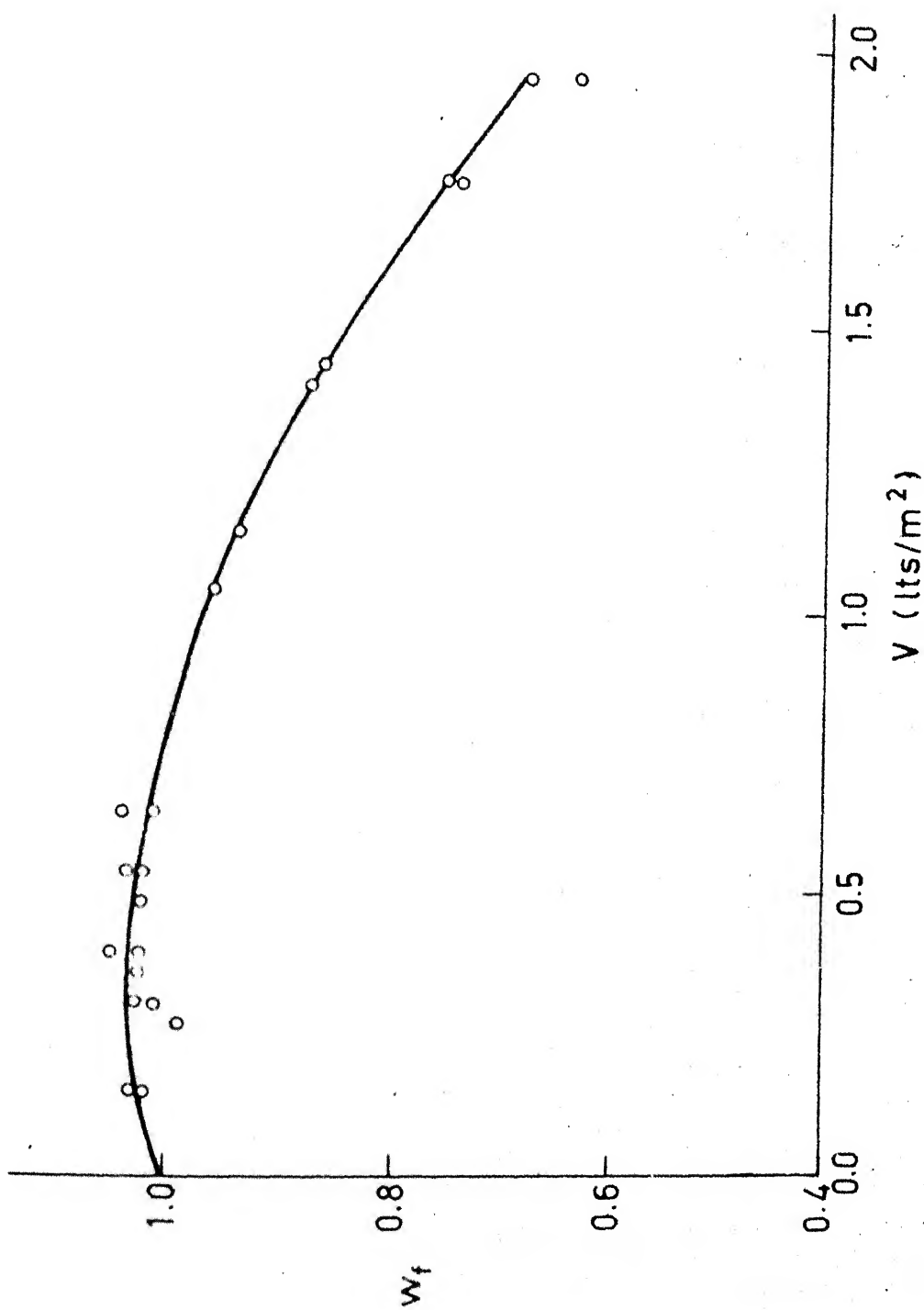


FIG. 3-17 VARIATION OF WEIGHT OF CONCRETE WITH
ACID CONSUMED

For ordinary concrete, a loss of weight, 33% in 30 weeks under 5 % exposure and 35% in 7 weeks under 15% exposure to sulphuric acid was reported by Glenn William DePuy [1] . The loss or gain of weight with the amount of acid consumed by concrete was fit in a second order polynomial. Equations (3.18) - (3.20) were used by replacing r_{fi}' by r_{wi}' and the respective V_i to solve for the polynomial constants.

$$\therefore r_w = \frac{W_{ce}}{W_{co}} = 1.011 + 0.0811 V - 0.128 V^2 \quad (3.24)$$

where W_{ce} - weight of exposed concrete

W_{co} - 28 days cube weight of concrete

r_w - weight reduction factor .

The percentage error at all points is of the order 1 to 10 %.

3.4.4 Depth of Deterioration of Concrete

Figure 3.18 shows the thickness of concrete dissolved with the amount of acid consumed. An equation of the type shown below is fit using the method of nonlinear regression.

Let

$$t_d = C_4 V^3 \quad (3.25)$$

where t_d is the depth of deterioration of concrete (thickness of concrete dissolved) and C_4 is a regression coefficient.

Then error at any point is $(nd) = (t_{di}' - t_{di})$

where t_{di}' is the i th observation (experimental).

The cumulative least square error is :

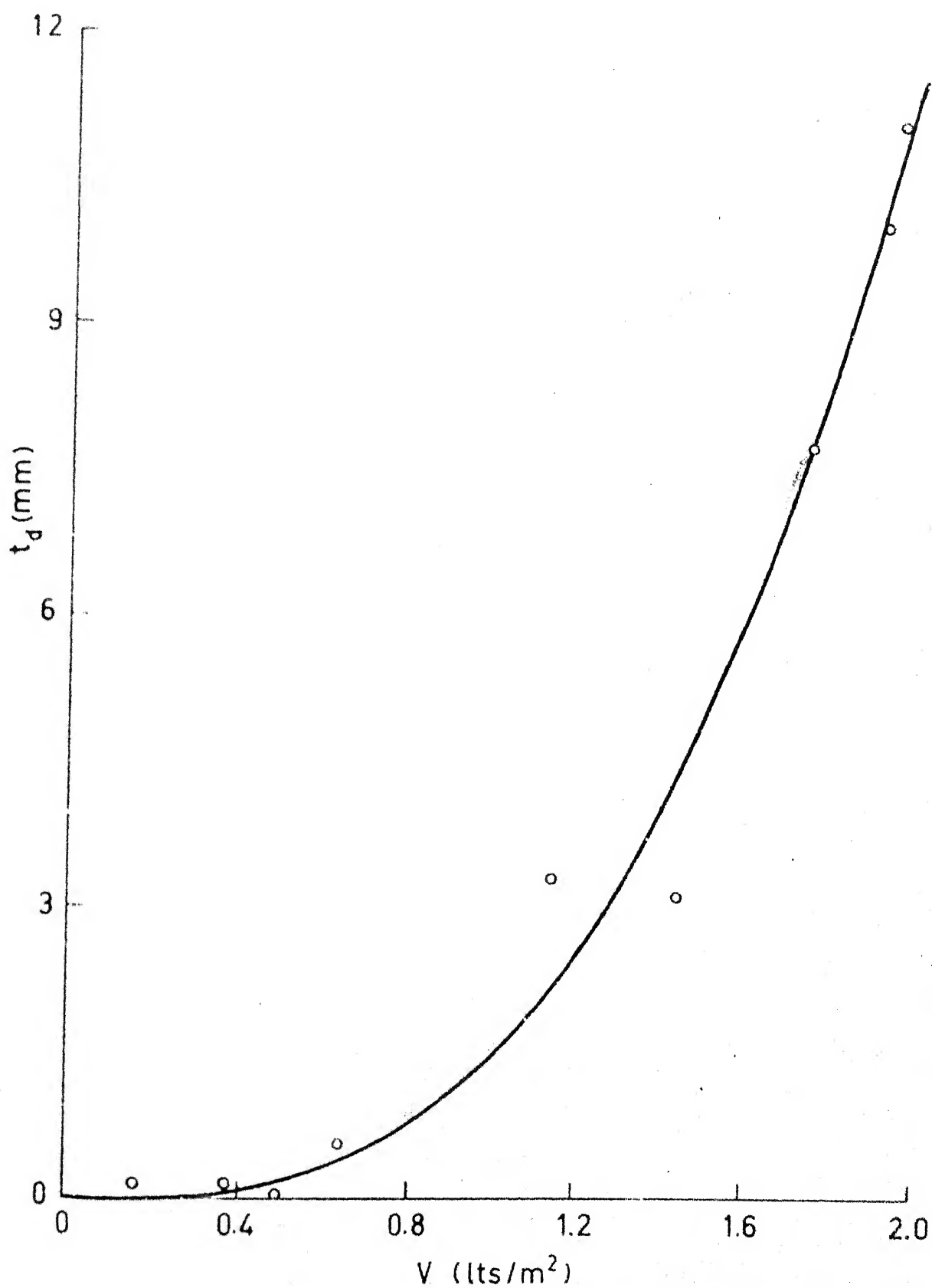


FIG 3.18 VARIATION OF DEPTH OF DETERIORATION OF CONCRETE WITH ACID CONSUMED.

If linear rate of reaction is assumed, Eq. (3.31) becomes

$\frac{q t_p}{2}$. For 1 m² area of concrete and t_p in mm, and $n = 1$

Eq. (3.31) becomes $\frac{q t_p}{2000}$ for linear rate of reaction. If

V l/m² of acid is consumed by the concrete and V_R is the volume of acid required for saturated reaction with one kN of cement, then cement reacted is $\frac{V}{V_R}$ kN/m².

$$\therefore \frac{q t_p}{2000} = \frac{V}{V_R}$$

$$\text{or } t_p = \frac{2000 V}{q V_R} \quad (3.32)$$

Figure 3.19 shows the variation of depth of penetration of acid (t_p) with the amount of acid consumed per unit surface area for different degrees of rate of reaction.

3.4.5 Coefficient of Variation of Exposed Concrete(Strength)

Figure 3.20 shows the variation of coefficient of variation of strength with the amount of acid consumed per unit surface area. It can be seen that the coefficient of variation is higher for higher amounts of acid consumed. Similarly, Fig. 3.21 shows the variation of coefficient of variation of strength with the strength of concrete. The coefficient of variation is higher for lower strengths. The reason for higher coefficient of variation may be due to

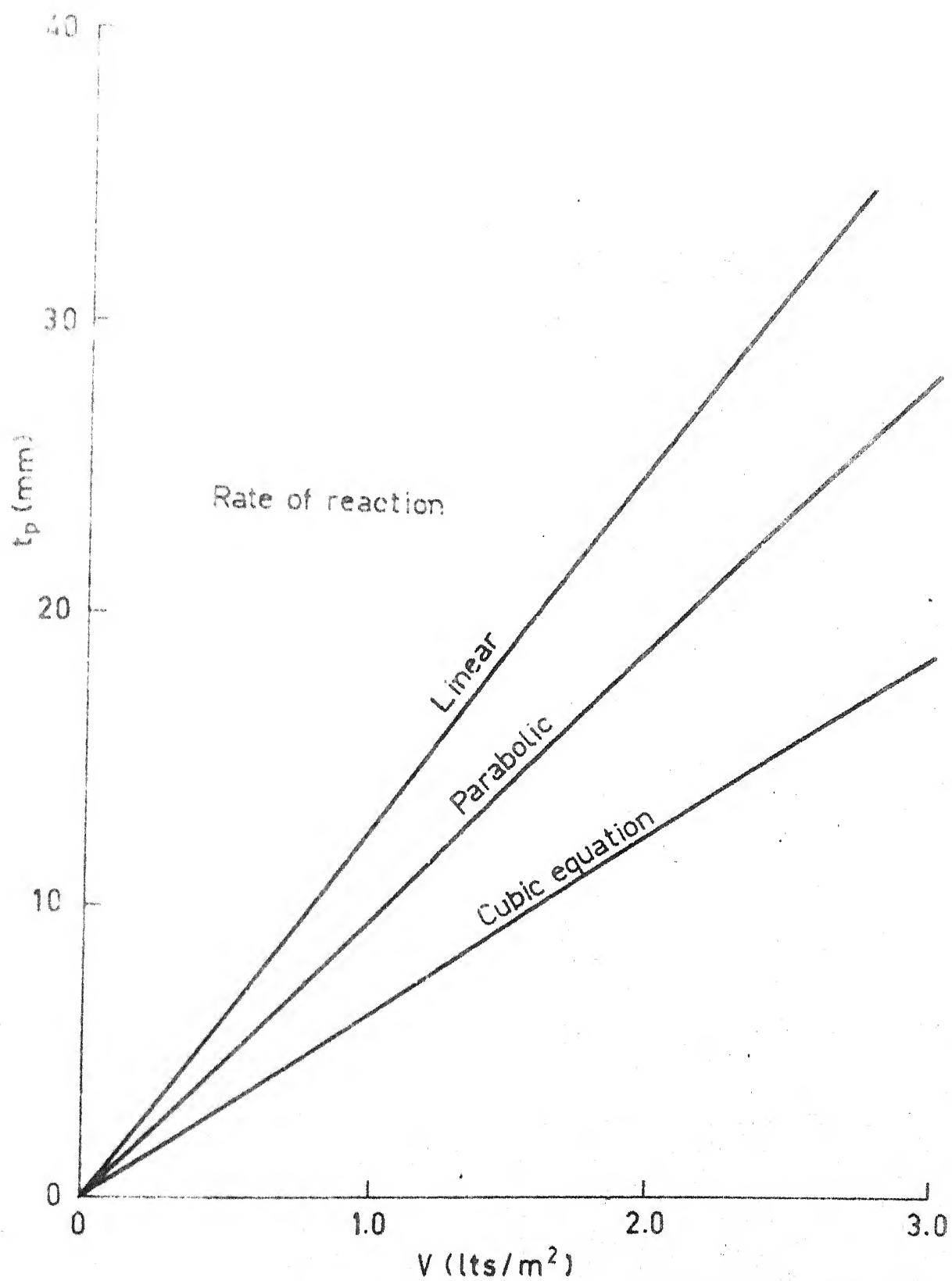


FIG.3.19 VARIATION OF DEPTH OF PENETRATION OF ACID WITH ACID CONSUMED

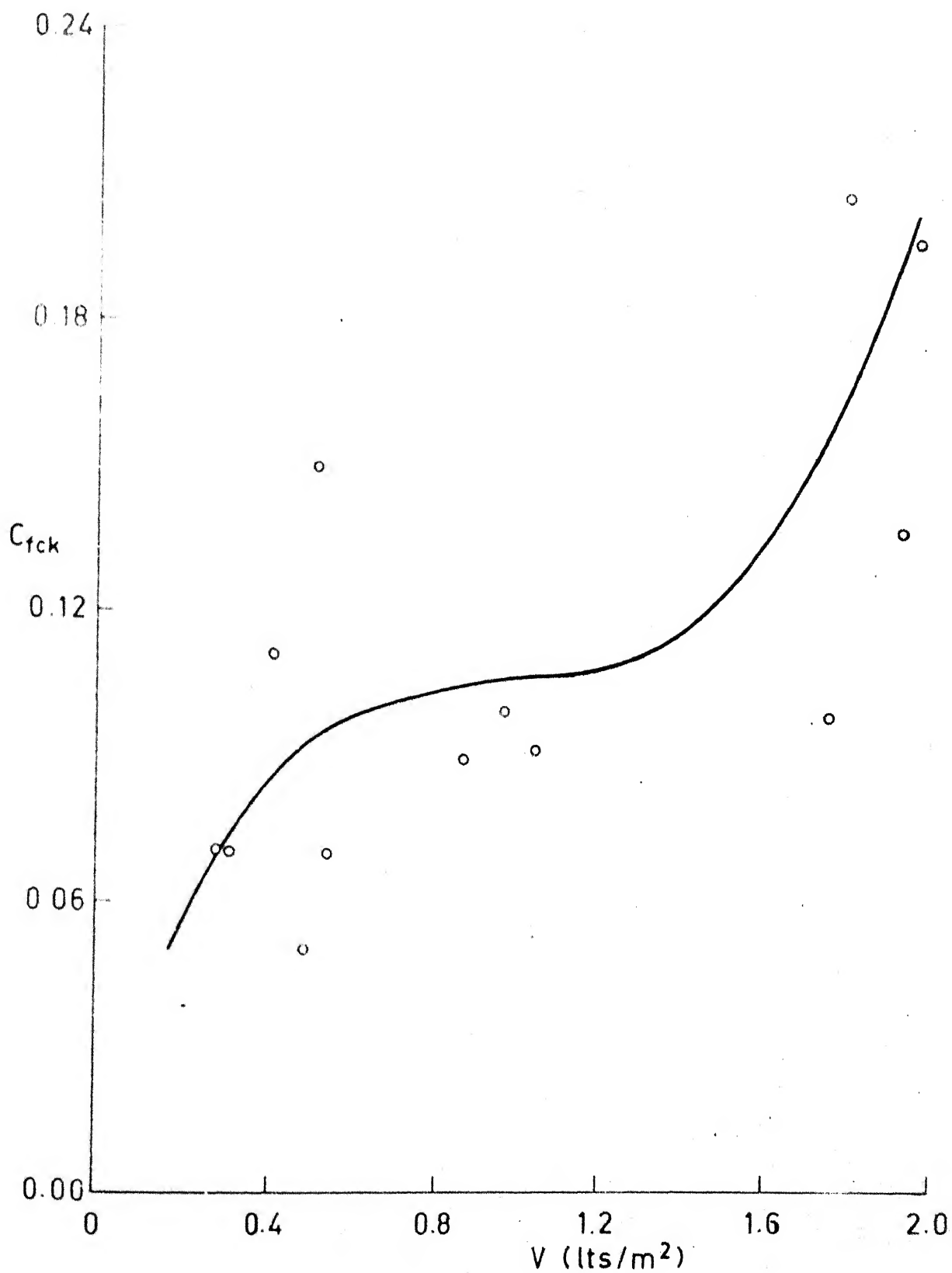


FIG. 3-20 VARIATION OF C_{fck} WITH ACID CONSUMED

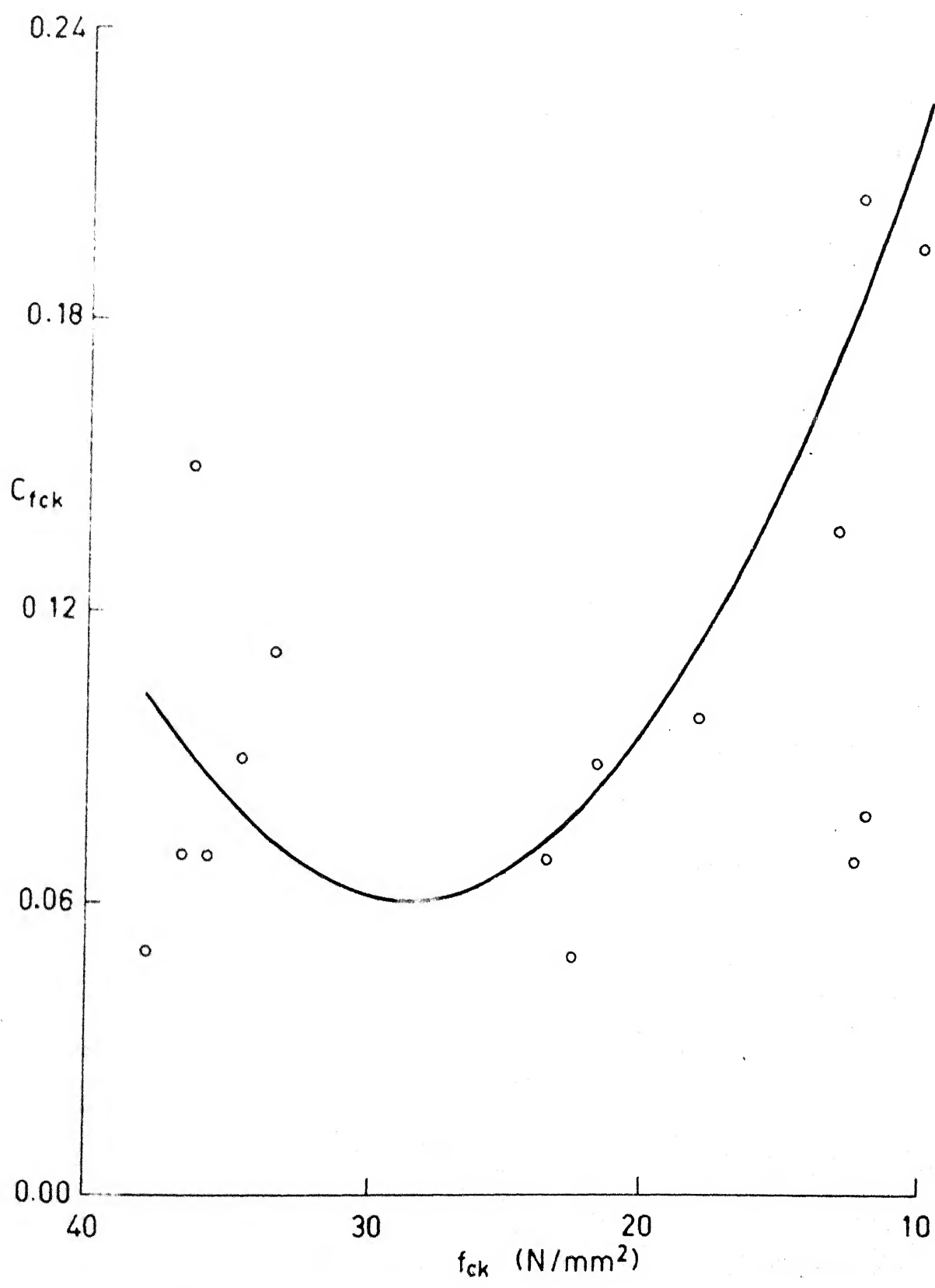


FIG. 3-21 VARIATION OF C_{fck} WITH STRENGTH OF CONCRETE

the unevenness of the concrete surface formed due to dissolving attack. Wherever the surface was not fully scaled off, a white cohesionless material was found on the surface (Figs. 3.2b and 3.3b) , which created lot of problems for capping. In such cases also, the coefficient of variation is high.

For the variation of coefficient of variation with the amount of acid consumed, a third order polynomial is fitted (Eq. (3.33)).

$$C_v = 0.01164 + 0.282 V - 0.2850 V^2 + 0.0982 V^3 \quad (3.33)$$

where C_v is the coefficient of variation of strength for the amount of acid consumed per unit surface area.

Similarly, for the variation of coefficient of variation of strength with the strength of concrete a second order polynomial is fitted.

$$C_{fck} = 0.4614 - 0.028 f_{co} + 0.000489 f_{co}^2 \quad (3.34)$$

where C_{fck} is the coefficient of variation of strength for a given strength of concrete, f_{co} is the strength of concrete in N/mm^2 .

The above observations can be used to estimate different parameters like: amount of acid consumed per unit surface area, depth of penetration of acid, strength and weight reduction factors, thickness of concrete dissolved and coefficient of variation of strength.

CHAPTER IV

DESIGN OF R C C BEAMS FOR DURABILITY CONSIDERATIONS

4.1 Introduction

Equations to estimate the strength reduction factor, deterioration of depth of concrete, depth of penetration of acid and coefficient of variation of strength of concrete have been developed in chapter three. This chapter presents a procedure for design of exposed reinforced concrete beams using the equations developed in the previous chapter, by both deterministic and probabilistic approaches. Design charts are presented and their use is illustrated through a set of examples. Costs of members, designed by different methods, are calculated for comparison purpose. The following rates are considered for calculating the cost of beams:

Concrete	Rs. 550/m ³
Steel	Rs. 600/kN
Form work	Rs. 50/m ² .

4.2 Variation of Moment of Resistance of an Exposed Beam

The moment of resistance of an exposed beam is normalized with the moment of resistance of an unexposed beam with the same properties. Figures are drawn to this ratio versus the amount of acid consumed for different percentages of tensile steel and different grades of steels.

Figures 4.1 - 4.16 are included at the end of the chapter.

4.2.1 Moment of Resistance of an Unexposed Beam

The neutral axis in a R C C rectangular section is given by [108]

$$\frac{x_u}{d} = \frac{0.87 f_y p_t}{36 f_{ck}} \quad (4.1)$$

$$k_b = \frac{305}{1265 + f_y} \quad (4.2)$$

where $\frac{x_u}{d}$ - ratio of the neutral axis depth to effective depth

f_y - characteristic yield strength of steel in N/mm^2

p_t - percentage of tensile steel

f_{ck} - characteristic compressive strength of concrete in N/mm^2 , and

k_b - ratio of the limiting neutral axis depth to effective depth.

Case I :

$$\text{If } f_{ck} < \frac{0.87 f_y p_t}{36 k_b}$$

then the moment capacity is given by

$$M_r = K k_b (1 - 0.42 k_b) \quad (4.3)$$

where M_r - moment of resistance

$$K = 0.36 f_{ck} b d^2$$

b - breadth of section

d - effective depth of section.

Case II :

$$\text{For } f_{ck} \geq \frac{0.87 f_y p_t}{36 k_b}$$

$$M_r = K \frac{x_u}{d} \left(1 - 0.42 \frac{x_u}{d} \right) \quad (4.4)$$

4.2.2 Moment of Resistance of an Exposed Beam

$$\frac{x_{ue}}{d} = \frac{0.87 f_y p_t}{36 f_{ck} r_f} \quad (4.5)$$

where $\frac{x_{ue}}{d}$ - ratio of neutral axis depth of an exposed section to effective depth.

Case I :

$$\text{For } f_{ck} r_f < \frac{0.87 f_y p_t}{36 k_b}$$

$$M_r(V) = K r_f k_b (1 - 0.42 k_b) \quad (4.6)$$

where $M_r(V)$ is the moment of resistance of an exposed beam.

Case II :

$$\text{For } f_{ck} r_f \geq \frac{0.87 f_y p_t}{36 k_b}$$

$$M_r(V) = K r_f \frac{x_{ue}}{d} \left(1 - 0.42 \frac{x_{ue}}{d} \right) \quad (4.7)$$

Now the ratio of moment of resistance of an exposed section to that of an unexposed one is calculated for different cases.

Case a :

$$\text{For } f_{ck} r_f < \frac{0.87 f_y p_t}{36 k_b} \text{ and } f_{ck} < \frac{0.87 f_y p_t}{36 k_b}$$

$$\frac{M_r(V)}{M_r} = \frac{r_f k_b (1 - 0.42 k_b)}{k_b (1 - 0.42 k_b)} = r_f \quad (4.8)$$

Case b :

$$\text{For } f_{ck} r_f < \frac{0.87 f_y p_t}{36 k_b} \text{ and } f_{ck} > \frac{0.87 f_y p_t}{36 k_b}$$

$$\frac{M_r(V)}{M_r} = \frac{r_f k_b (1 - 0.42 k_b)}{\frac{x_u}{d} (1 - 0.42 \frac{x_u}{d})} \quad (4.9)$$

Case c :

$$\text{For } f_{ck} r_f > \frac{0.87 f_y p_t}{36 k_b} \text{ and } f_{ck} < \frac{0.87 f_y p_t}{36 k_b}$$

This case does not occur, because $f_{ck} r_f$ is less than f_{ck} .

Case d :

$$f_{ck} r_f > \frac{0.87 f_y p_t}{36 k_b} \text{ and } f_{ck} > \frac{0.87 f_y p_t}{36 k_b}$$

$$\frac{M_r(V)}{M_r} = \frac{r_f \frac{x_{ue}}{d} (1 - 0.42 \frac{x_{ue}}{d})}{\frac{x_u}{d} (1 - 0.42 \frac{x_u}{d})} \quad (4.10)$$

From equations (4.1) and (4.5) $\frac{x_u}{d} = \frac{x_{ue}}{d} r_f$

$$\frac{M_r(V)}{M_r} = \frac{(1 - 0.42 \frac{x_{ue}}{d})}{(1 - 0.42 \frac{x_u}{d})} \quad (4.11)$$

For a given set of values of f_{ck} , f_y , p_t and V , one can estimate the ratio of moment of resistance of an exposed beam to that of an unexposed one, using the appropriate equations from Eqs. (4.8) - (4.11). The statement, $f_{ck} r_f$ less than f_{ck} is not true for very small values of V , where the value of r_f is slightly more than one. However in the normal working level, the value of r_f is less than one.

4.3 Deterministic Design of an Exposed Beam

If the concentration of sulphuric acid to which the concrete (W/C ratio 0.45 and A/C ratio 4.5) exposed is known, one can estimate the strength reduction factor (r_f) from Eq. (3.21) and depth of penetration of acid (t_p) from Eq. (3.32). For calculating these two factors, the constants B and C are required and they can be obtained from Fig. 3.13. After obtaining the values of B and C, a value V , the acid consumed per unit surface area, is to be calculated, which will be finally used in estimating r_f and t_p .

$$\frac{x_{ue}}{d} = \frac{0.87 f_y p_t}{36 f_{ck} r_f} \quad (4.5)$$

$$M_r = 0.36 f_{ck} r_f b d^2 \frac{x_{ue}}{d} \left(1 - 0.42 \frac{x_{ue}}{d} \right) \quad (4.12)$$

$\frac{x_{ue}}{d}$ should be replaced by k_b , if $\frac{x_{ue}}{d} > k_b$.

Assuming a value of width of a beam, the depth d can be estimated using Eqs. (3.21) and (4.12), for the specified M_R , f_{ck} , f_y , p_t and V . Similarly the design of an unexposed beam can be done either by using appropriate equation from (4.3) and (4.4) or any standard design tables.

4.4 Estimation of Coefficient of Variation of Moment of Resistance

The mean value and coefficient of variation of M_R are needed in probabilistic design of members, which is presented in the next section. Normally the design parameters are treated as random variables, since the variability in the parameters is unavoidable. The underlying probability distribution of the random variables is assumed as Gaussian. The mean and standard deviation of a function can be calculated using the mean and standard deviation of the random variables [107] .

$$\bar{\phi} = \phi(\bar{v}_1, \bar{v}_2, \dots, \bar{v}_n) \quad (4.13)$$

$$s_{\phi} = \left\{ \sum_{i=1}^n \left[\frac{\partial \phi}{\partial v_i} \mid (\bar{v}_1, \bar{v}_2, \dots, \bar{v}_n) \right]^2 s_{vi}^2 \right\}^{\frac{1}{2}} \quad (4.14)$$

where ϕ - function of random variables $v_1, v_2 \dots v_n$

s_{ϕ} - standard deviation of a function of random variables
 $v_1, v_2 \dots v_n$

Bar over variable indicates the mean of that variable.

In further use, the bar is dropped for the sake of convenience.

$$\text{Mean of } \frac{x_u}{d} = \frac{0.37 f_y A_{st}}{0.36 f_{ck} b d} \quad (4.15)$$

$$\text{Standard deviation of } \frac{x_u}{d} = s_{xud} = \sqrt{(\text{variance})}$$

Using Eq. (4.14)

$$\begin{aligned} s_{xud}^2 = & \left(\frac{0.37 A_{st}}{0.36 f_{ck} b d} \right)^2 s_{fy}^2 + \left(\frac{0.87 f_y}{0.36 f_{ck} b d} \right)^2 s_{A_{st}}^2 \\ & + \left(\frac{0.37 f_y A_{st}}{0.36 f_{ck}^2 b d} \right)^2 s_{f_{ck}}^2 + \left(\frac{0.37 f_y A_{st}}{0.36 f_{ck} b^2 d} \right)^2 s_b^2 \\ & + \left(\frac{0.87 f_y A_{st}}{0.36 f_{ck} b d^2} \right)^2 s_d^2 \end{aligned} \quad (4.16)$$

Replacing the standard deviation by the corresponding product of mean and coefficient of variation, Eq. (4.16) reduces to

$$s_{xud}^2 = \left(\frac{0.37 f_y A_{st}}{0.36 f_{ck} b d} \right)^2 (C_{fy}^2 + C_{A_{st}}^2 + C_{f_{ck}}^2 + C_b^2 + C_d^2) \quad (4.17)$$

where C is the coefficient of variation of a random variable, with the variable written as its subscript.

$$\therefore C_{xud} = \frac{s_{xud}}{x_u/d} = (C_{fy}^2 + C_{A_{st}}^2 + C_{f_{ck}}^2 + C_b^2 + C_d^2)^{\frac{1}{2}} \quad (4.18)$$

Case I :

$$\text{For } \frac{x_u}{d} > k_b$$

$$M_r = 0.36 f_{ck} b d^2 k_b (1 - 0.42 k_b) \quad (4.19)$$

$$\text{Variance of } M_r = s_{Mr}^2$$

$$= M_r^2 (c_{fck}^2 + c_b^2 + 4 c_d^2) \quad (4.20)$$

$$\text{Coefficient of variation of } M_r = C_{Mr}$$

$$= \frac{s_{Mr}}{M_r} = (c_{fck}^2 + c_b^2 + c_d^2)^{\frac{1}{2}} \quad (4.21)$$

Case II:

$$\text{For } \frac{x_u}{d} < k_b$$

$$M_r = 0.87 f_y A_{st} d (1 - 0.42 \frac{x_u}{d}) \quad (4.22)$$

$$s_{Mr}^2 = M_r^2 (c_{fy}^2 + c_{Ast}^2 + c_d^2) + M_r^2 \left(\frac{0.42 \frac{x_u}{d} c_{xud}}{1 - 0.42 \frac{x_u}{d}} \right)^2 \quad (4.23)$$

$$C_{Mr} = \frac{s_{Mr}}{M_r}$$

$$= [c_{fy}^2 + c_{Ast}^2 + c_d^2 + \left(\frac{0.42 \frac{x_u}{d} c_{xud}}{1 - 0.42 \frac{x_u}{d}} \right)^2]^{\frac{1}{2}} \quad (4.24)$$

4.5 Probabilistic Design of an Exposed Beam

Probabilistic design approach has already gained the importance in engineering design. It is desirable that the

members should be proportioned keeping in view of the variabilities in the parameters. It has already been discussed in the previous section about the estimation of coefficient of variation of moment of resistance, if the variability of the parameters is specified. The moment of resistance of the section and external moment are assumed to follow Gaussian distribution.

The probability of failure can be estimated as follows:

$$Z = \frac{M_e - M_r}{\sqrt{s_{Me}^2 + s_{Mr}^2}} \quad (4.25)$$

where M_e is an external moment.

Replacing standard deviation by mean and coefficient of variation and rearranging the terms Eq. (4.25) reduces to

$$Z = \frac{1 - \frac{M_r}{M_e}}{\sqrt{(\alpha_{Me}^2 + \alpha_{Mr}^2 \frac{M_r^2}{M_e^2})}} \quad (4.26)$$

Probability of failure = P_f

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^Z e^{-x^2/2} dx = \Phi(Z)$$

where Φ is a standard normal probability function

$$\therefore Z = \Phi^{-1}(P_f) \quad (4.27)$$

Rearranging Eq. (4.26)

$$\frac{M_r}{M_e} = \frac{1 \pm \sqrt{[1 - (1 - Z^2 C_{Me}^2) (1 - Z^2 C_{Mr}^2)]}}{(1 - Z^2 C_{Mr}^2)} \quad (4.28)$$

So for a given probability of failure and coefficient of variation of external moment and moment capacity, the ratio M_r/M_e can be calculated using Eqs. (4.27) and (4.28). Procedure given in Appendix B can be used for proportioning the section after finalizing the design moment.

Rearranging Eq. (4.26) :

$$C_{Mr} = \left[\frac{(1 - M_r/M_e)^2}{Z^2} - C_{Me}^2 \right]^{-\frac{1}{2}} \quad (4.29)$$

$$C_{Me} = \left[\frac{(1 - M_r/M_e)^2}{Z^2} - \frac{M_r^2}{M_e^2} - C_{Mr}^2 \right]^{-\frac{1}{2}} \quad (4.30)$$

For probability of failure 10^{-5} , $Z = -4.265$, the corresponding maximum value of C_{Me} is 0.117 for $C_{Mr} = 0$ (from Eq. (4.30)). Similarly, the maximum value of $C_{Mr} = 0.073$ for $C_{Me} = 0$ (from Eq. (4.29)).

If the value under the square root in Eq. (4.28) is less than zero, the solution is absurd, because M_r/M_e can never be a complex quantity. Therefore this puts a limit on C_{Me} , C_{Mr} and Z for the probability of failure to exist.

i.e. $1 - (1 - Z^2 C_{Me}^2)(1 - Z^2 C_{Mr}^2) \geq 0$ for the existence of a real solution.

$$\therefore Z^2 C_{Me}^2 \leq 1 \text{ and } Z^2 C_{Mr}^2 \leq 1 \quad (4.31)$$

$$\text{and } (1 - Z^2 C_{Me}^2)(1 - Z^2 C_{Mr}^2) \leq 1 \quad (4.32)$$

At the same time, the denominator of Eq. (4.28) should also be satisfied. That is $(1 - Z^2 C_{Mr}^2) > 0$ always, because for a real solution to exist the value under the square root is always less than 1. If the denominator is negative, M_r/M_e becomes negative. Similarly, if denominator is equal to zero, M_r/M_e becomes infinite. Both cases are absurd.

$$\therefore Z^2 C_{Mr}^2 < 1$$

Eq. (4.31) is modified as

$$Z^2 C_{Me}^2 \leq 1 \text{ and } Z^2 C_{Mr}^2 < 1$$

For example, let probability of failure be 10^{-5} , for which Z is -4.265 .

$$\therefore C_{Mr}^2 < \frac{1}{(-4.265)^2}$$

$$\text{or } C_{Mr} < 0.234 \text{ and } C_{Me} \leq 0.234 .$$

This is only a limit on C_{Mr} and C_{Me} . Actually, Z , C_{Mr} and C_{Me} put together should also satisfy Eq. (4.32). So Eqs. (4.31) - (4.33) can be used to check whether a member

can be designed for the required probability of failure with the given variabilities.

4.6 Examples

Example 4.1.:

A rectangular beam designed to resist an external moment 180 kNm, is exposed to sulphuric acid of 0.1 % concentration. $f_y = 415 \text{ N/mm}^2$, $f_{ck} = 20 \text{ N/mm}^2$ and $p_t = 0.4$. Determine the moment of resistance of the beam at the end of 20 years.

$$V = 30 [1 - \exp(-Bt^C)] \quad (3.5)$$

$$B = 0.01882 \quad \left\{ \begin{array}{l} \text{from Fig. 3.13} \\ \text{from Fig. 3.13} \end{array} \right.$$

$$C = 0.50$$

$$t = 20 \text{ years}$$

$$V = 30 [1 - \exp(-0.01882 \times \sqrt{20})] = 2.42 \text{ l/m}^2$$

$$\begin{aligned} r_f &= 1.14 - 0.5741 V + 0.1163 V^2 \quad (3.21) \\ &= 1.14 - 0.5741 \times 2.42 + 0.1163 \times 2.42^2 = 0.432 \end{aligned}$$

$$f_{ck} r_f = 20 \times 0.432 = 8.64 \text{ N/mm}^2$$

$$\frac{0.37 f_y p_t}{36 k_b} = \frac{0.37 \times 415 \times 0.4}{36 \times 0.43} = 8.36$$

where $k_b = 0.48$ from [109]

$$f_{ck} > f_{ck} r_f > \frac{0.37 f_y p_t}{36 k_b}$$

Using Eq. (4.11)

$$\frac{M_r(2.42)}{M_r} = \frac{\left(1 - \frac{0.42 \times 0.37 \times 415 \times 0.4}{36 \times 20 \times 0.432}\right)}{\left(1 - \frac{0.42 \times 0.87 \times 415 \times 0.4}{36 \times 20}\right)} = 0.879$$

Alternatively, from Fig. 4.2

$$\frac{M_r(V)}{M_r} = \frac{M_r(2.42)}{M_r} = 0.88$$

$$M_r(2.42) = 0.88 \times 180 = 158.4 \text{ kN m}$$

Example 4.2

(a) Design a reinforced concrete rectangular beam for an external moment of 120 kNm to survive for 20 years in a sulphuric acid of 0.1 % concentration. The concrete is made with A/C ratio 4.5 and W/C ratio 0.45. $f_y = 415 \text{ N/mm}^2$, $q = 4.26 \text{ kN/m}^3$, and $V_R = 73.4 \text{ l/kN}$.

(b) If the depth of penetration of acid is limited to 20 mm, estimate the minimum cement content required.

(c) The cover provided is 25 mm. Estimate the time taken for the acid to penetrate 25 mm cover.

(d) Find out the depth of deterioration of concrete.

$$\begin{aligned} V &= 2.42 \text{ l/m}^2 & \text{from example 4.1} \\ r_f &= 0.432 \end{aligned}$$

(e) Let $p_t = 0.4$

$$\frac{x_{ue}}{d} = \frac{0.87 \times 415 \times 0.4}{36 \times 20 \times 0.432} = 0.464 \text{ from Eq. (4.5)}$$

$$\frac{x_{ue}}{d} < k_b$$

$$\therefore M_r = 0.36 f_{ck} r_f b d^2 \frac{x_{ue}}{d} (1 - 0.42 \frac{x_{ue}}{d}) \quad (4.12)$$

$$= 0.36 \times 20 \times 0.432 \times b \times d^2 \times 0.464 (1 - 0.42 \times 0.464)$$

$$= 180 \times 10^6 \text{ N mm}$$

Let $b = 2/3 d$, then the above equation yields:

$$d = 615 \text{ mm}$$

$$b = \frac{2}{3} \times 615 = 410 \text{ mm}$$

$$A_{st} = \frac{0.4 \times 612 \times 403}{100} = 1008 \text{ mm}^2$$

Alternatively, from Fig. 4.2

$$\frac{M_r(2.42)}{M_r} = 0.88$$

$$M_r = \frac{180}{0.88} = 205 \text{ kN m}$$

From Fig. 4.13

$$\frac{M_r}{bd^2} = 1.32 \text{ for } p_t = 0.4$$

Let $b = 2/3 d$

$$d = \sqrt[3]{(205 \times 1.5 \times 10^6 / 1.32)} = 614 \text{ mm}$$

$$b = 410 \text{ mm}$$

$$A_{st} = 1006 \text{ mm}^2$$

Provide $b = 410 \text{ mm}$

$D = 670 \text{ mm}$, where D is overall depth

3 bars of $20 \text{ mm } \phi$ and 1 bar of $10 \text{ mm } \phi$

$$\therefore A_{st} = 1021 \text{ mm}^2$$

Details of the section are shown in Fig. 4.14b .

If the beam is unexposed:

$$M_r = 180 \text{ kNm} ; p_t = 0.4 ; b = 2/3 d$$

From Fig. 4.13

$$\frac{M_r}{bd^2} = 1.32$$

$$d = 588 \text{ mm}$$

$$b = 390 \text{ mm}$$

$$A_{st} = 918 \text{ mm}^2$$

$$\text{Provide } b = 390 \text{ mm}$$

$$D = 640 \text{ mm}$$

3 bars of $20 \text{ mm } \phi$

$$\therefore A_{st} = 943 \text{ mm}^2$$

Details of the section are shown in Fig. 4.14a.

Cost of unexposed beam/m length:

$$\text{Cost of concrete} = 0.39 \times 0.64 \times 550 = \text{Rs. } 137$$

$$\text{Cost of steel} = 77 \times 943 \times 10^{-6} \times 600 = \text{Rs. } 43$$

$$\text{Cost of form work} = (0.39 + 2 \times 0.64) 50 = \text{Rs. } 84$$

$$\text{Total cost} = \text{Rs. } 264$$

Cost of exposed beam/m length:

$$\text{Cost of concrete} = 0.41 \times 0.67 \times 550 = \text{Rs. } 151$$

$$\text{Cost of steel} = 77 \times 1021 \times 10^{-6} \times 600 = \text{Rs. } 47$$

$$\text{Cost of form work} = (0.41 + 2 \times 0.67) 50 = \text{Rs. } 88$$

$$\text{Total Cost} = \text{Rs. } 286$$

$$(b) \quad q = \frac{2000 V}{t_p V_R} = \frac{2000 \times 2.42}{20 \times 73.4} = 3.3 \text{ kw/m}^3.$$

$$(B) \quad t_p = 25 \text{ mm}$$

$$V = \frac{q t_p V_R}{2000} \quad \text{from Eq. (3.32)}$$

$$\therefore V = \frac{4.26 \times 25 \times 73.4}{2000} = 3.91 \text{ l/m}^2$$

$$V = 30 [1 - \exp(-0.01882 \sqrt{t})] = 3.91 \text{ l/m}^2$$

Solving we get $t = 55 \text{ years}$

$$(d) \quad t_d = 1.47 V^3 \quad (3.29)$$

$$V = 2.42 \text{ l/m}^2$$

$$t_d = 21 \text{ mm}$$

TABLE 4.1 : DESIGN VARIABLES AND COST OF A BEAM
(DETERMINISTIC APPROACH)

Variable	Unexposed	Exposed
b (mm)	390	410
D (mm)	640	670
A _{st} (mm ²)	943	1021
Cost/m (Rs.)	264	286

Example 4.3

Design a R C rectangular beam for an external moment of 120 kN m. The beam will be exposed to sulphuric acid of 0.01 concentration. It should survive for 20 years with a probability of failure: a) 10^{-4} , b) 10^{-5} , c) 10^{-6} , d) 10^{-7} and e) 10^{-8} . The coefficient of variation of external moment is 0.15, and that of steel is 0.05. Take $p_t = 0.3$, $f_{ck} = 20 \text{ N/mm}^2$ and $f_y = 415 \text{ N/mm}^2$.

$$A = 30 ; B = 0.007 \text{ and } C = 0.5 \text{ from Fig. 3.13.}$$

$$V = 30 [1 - \exp(-0.007 \sqrt{20})] = 0.925 \text{ l/m}^2$$

$$r_f = 1.14 - 0.574 \times 0.925 + 0.1163 \times 0.925^2 = 0.71$$

$$C_{fck} = 0.4614 - 0.028 f_{ck} + 0.000489 f_{ck}^2 \quad (3.34)$$

$$= 0.4614 - 0.028 \times 0.71 \times 20 + 489 \times 10^{-6} \times 400 \times 0.71^2 = 0.163$$

From Eqs. (B.4) and (B.5)

$$\frac{x_{ue}}{d} = \frac{415 \times 0.3}{0.543 \times 20 \times 0.71 \times 100} = 0.431 < k_b$$

$$C_{xued} = \sqrt{C_{fy}^2 + C_{Ast}^2 + C_{fck}^2 + C_b^2 + C_d^2} \quad (4.18)$$

$$= \sqrt{0.05^2 + 0.163^2} = 0.170$$

$$C_{Mr} = \left[C_{fy}^2 + C_{Ast}^2 + C_d^2 + \left(\frac{0.42 \frac{x_{ue}}{d} C_{xued}}{1 - 0.42 \frac{x_{ue}}{d}} \right)^2 \right]^{\frac{1}{2}} \quad (4.24)$$

$$= \left[0.05^2 + \left(\frac{0.42 \times 0.431 \times 0.17}{1 - 0.42 \times 0.431} \right)^2 \right]^{\frac{1}{2}} = 0.063$$

$$(a) \quad P_f = 10^{-4} ; \quad z = \Phi^{-1}(10^{-4}) = -3.72$$

$$C_{Me} = 0.15 ; \quad C_{Mr} = 0.063.$$

$$\frac{M_r}{M_e} = \frac{1 \pm \sqrt{[1 - (1 - z^2 C_{Me}^2)(1 - z^2 C_{Mr}^2)]}}{(1 - z^2 C_{Mr}^2)} \quad (4.23)$$

$$= \frac{1 \pm \sqrt{(1 - 0.688 \times 0.945)}}{0.945} = 1.69$$

Alternatively, from Fig. 4.11 $M_r/M_e = 1.69$ for the above details.

$$\therefore \text{Design moment} = 1.69 \times 120 = 203 \text{ kN m}$$

$$\text{Let } b = \frac{2}{3} d$$

$$M_r = 0.543 f_{ck} r_f \frac{2}{3} d^3 \cdot 0.431 (1 - 0.42 \times 0.431)$$

$$= 1.8135 d^3 = 203 \times 10^6 \text{ N mm}$$

$$\therefore d = 432 \text{ mm} ; \quad b = 322 \text{ mm} ; \quad A_{st} = 1242 \text{ mm}^2$$

Provide $b = 325 \text{ mm}$; $D = 535 \text{ mm}$;

$$4 \text{ nos. } 20 \text{ mm } \phi ; A_{st} = 1256 \text{ mm}^2$$

Details of section are shown in Fig. 4.15d.

Cost of beam/m length:

$$\text{Cost of concrete} = 0.325 \times 0.535 \times 550 = \text{Rs. } 96$$

$$\text{Cost of steel} = 77 \times 1256 \times 10^{-6} \times 600 = \text{Rs. } 58$$

$$\text{Cost of form work} = (0.325 + 2 \times 0.535) 50 = \text{Rs. } 70$$

$$\text{Total cost} = \text{Rs. } 224$$

$$(b) \quad P_f = 10^{-5}$$

From Fig. 4.11

$$\frac{M_r}{M_e} = 1.308 \text{ for } C_{Me} = 0.15 \text{ and } C_{Mr} = 0.063$$

$$\text{Design moment} = 1.308 \times 120 = 216.24 \text{ kN m}$$

$$\text{Let } b = 2/3 d$$

$$d = \sqrt[3]{(216.24 \times 10^6 / 1.3135)} = 493 \text{ mm}$$

$$\therefore b = 323 \text{ mm} ; A_{st} = 1294 \text{ mm}^2$$

Provide $b = 330 \text{ mm}$; $D = 550 \text{ mm}$.

4 nos. $20 \text{ mm } \phi$ bars + 1 no. $10 \text{ mm } \phi$ bar

$$\therefore A_{st} = 1335 \text{ mm}^2$$

Details are shown in Fig. 4.15e.

Cost of beam/m length:

Cost of concrete	=	Rs. 100
Cost of steel	=	Rs. 62
Cost of form work	=	Rs. 72

Total cost Rs. 234

$$(c) \quad P_f = 10^{-6}$$

$$\frac{M_p}{M_o} = 1.924 \quad \text{from Fig. 4.11}$$

$$\text{Design moment} = 1.924 \times 120 = 230.82 \text{ kN m}$$

$$\text{Let } b = 2/3 d$$

$$d = \sqrt[3]{(230.82 \times 10^6 / 1.3135)} = 503 \text{ mm}$$

$$\therefore b = 335 \text{ mm}; \quad A_{st} = 1350 \text{ mm}^2$$

$$\text{Provide } b = 335 \text{ mm}; \quad D = 560 \text{ mm}$$

$$3 \text{ nos. } 20 \text{ mm } \phi \text{ bars} + 4 \text{ nos } 12 \text{ mm } \phi \text{ bars}$$

$$\therefore A_{st} = 1395 \text{ mm}^2$$

Details of the section are shown in Fig. 4.15f.

Cost of beam/m length:

Cost of concrete	=	Rs. 103
Cost of steel	=	Rs. 64
Cost of form work	=	Rs. 73

Total cost Rs. 240

$$(d) \quad P_f = 10^{-7}$$

From Fig. 4.11 $M_r/M_e = 2.035$

$$M_r = 2.035 \times 120 = 245 \text{ kN m}$$

Let $b = 2/3 d$

$$d = \sqrt[3]{(245 \times 10^6 / 1.8135)} = 513 \text{ mm}$$

$$\therefore b = 342 \text{ mm} ; A_{st} = 1404 \text{ mm}^2$$

Provide $b = 345 \text{ mm} ; D = 570 \text{ mm}$

4 nos. 20 mm ϕ bars + 2 nos. 10 mm ϕ bars

$$\therefore A_{st} = 1413 \text{ mm}^2$$

Details of the sections are shown in Fig. 4.15g.

Cost of beam/m length:

Cost of concrete = Rs. 103

Cost of steel = Rs. 65

Cost of form work = Rs. 74

Total cost Rs. 247

$$(e) \quad P_f = 10^{-8}$$

From Fig. 4.11 $M_r/M_e = 2.15$

$$M_r = 2.15 \times 120 = 258 \text{ kN m}$$

Let $b = 2/3 d$

$$d = \sqrt[3]{(258 \times 10^6 / 1.8135)} = 522 \text{ mm}$$

$$\therefore b = 348 \text{ mm} ; A_{st} = 1453 \text{ mm}^2$$

Provide $d = 350 \text{ mm}$; $D = 575 \text{ mm}$

4 nos. 20 mm \emptyset bars + 1 no. 16 mm \emptyset bars

$$\therefore A_{st} = 1457 \text{ mm}^2$$

Details of the sections are shown in Fig. 4.15h.

Cost of beam/m length:

Cost of concrete = Rs. 111

Cost of steel = Rs. 67

Cost of form work = Rs. 75

Total Cost Rs. 253

For comparison purpose, deterministic design of a) exposed beam , b) unexposed beam and c) probabilistic design of unexposed beam are carried out as follows. The other details are same as given in Example 4.3.

$$(a) \quad M_r = 120 \times 1.5 = 180 \text{ kN m}$$

$$\text{From Fig. 4.2 , } M_r(0.925) / M_r = 0.815$$

$$\therefore \text{Design moment} = \frac{180}{0.815} = 221 \text{ kN m}$$

$$\text{From Fig. 4.13, } \frac{M_u}{bd^2} = 2.40 \text{ for } p_t = 0.8$$

$$\text{Let } b = 2/3 d$$

$$d = \sqrt[3]{(221 \times 10^6 \times 1.5) / 2.40} = 517 \text{ mm}$$

$$\therefore b = 344 \text{ mm} ; A_{st} = 1423 \text{ mm}^2$$

Provide $b = 350 \text{ mm}$; $D = 570 \text{ mm}$

4 Nos. 20 mm ϕ bars + 1 no. 16 mm ϕ bar

$$\therefore A_{st} = 1457 \text{ mm}^2$$

Details of the section are shown in Fig. 4.15b.

Cost of beam/m length:

Cost of concrete = Rs. 110

Cost of steel = Rs. 67

Cost of form work = Rs. 74

Total Cost Rs. 251

(b) $M_r = 120 \times 1.5 = 180 \text{ kN m}$; Let $b = 2/3 d$

From Fig. 4.13 $\frac{M_u}{bd^2} = 2.40$ for $p_t = 0.8$

$$d = \sqrt[3]{(180 \times 10^6 \times 1.5 / 2.40)} = 482 \text{ mm}$$

$$\therefore b = 321 \text{ mm} ; A_{st} = 1243 \text{ mm}^2$$

Provide $b = 325 \text{ mm}$; $D = 540 \text{ mm}$

4 Nos. 20 mm ϕ bars ; $A_{st} = 1256 \text{ mm}^2$

Details are as shown in Fig. 4.15a.

Cost of beam/m length:

Cost of concrete = Rs. 97

Cost of steel = Rs. 58

Cost of form work = Rs. 70

Total cost Rs. 225

(c) Probabilistic design of unexposed beam

$$P_f = 10^{-5} ; C_{Me} = 0.15$$

$$C_{fek} = 0.4614 - 0.028 \times 20 + 0.489 \times 10^{-3} \times 400 = 0.10$$

$$\frac{x_u}{d} = \frac{415 \times 0.3}{0.543 \times 20 \times 100} = 0.306 < k_b$$

$$C_{xud} = \sqrt{(0.1)^2 + (0.05)^2} = 0.112$$

$$C_{Me} = \left[0.05^2 + \left(\frac{0.42 \times 0.306 \times 0.112}{1 - 0.42 \times 0.306} \right)^2 \right]^{\frac{1}{2}} = 0.053$$

$$\frac{M_r}{M_e} = 1.760 \quad \text{from Fig. 4.11}$$

$$M_r = 1.760 \times 120 = 211.2 \text{ kN m}$$

$$\begin{aligned} M_r &= 0.5427 \times 20 \times \frac{2}{3} d^3 \times 0.306 (1 - 0.42 \times 0.306) \\ &= 211.2 \times 10^6 \text{ N mm} \end{aligned}$$

$$d = \sqrt[3]{(211.2 \times 10^6 / 1.93)} = 479 \text{ mm}$$

$$\therefore b = 319 \text{ mm} ; A_{st} = 1224 \text{ mm}^2$$

$$\text{Provide } b = 320 \text{ mm} ; D = 530 \text{ mm}$$

$$4 \text{ Nos. of } 20 \text{ mm } \emptyset \text{ bars} ; A_{st} = 1256 \text{ mm}^2.$$

Details are shown in Fig. 4.15c.

Cost of beam /m length:

$$\text{Cost of concrete} = \text{Rs. } 94$$

$$\text{Cost of steel} = \text{Rs. } 53$$

$$\text{Cost of Form work} = \text{Rs. } 69$$

$$\text{Total Cost} = \text{Rs. } 221$$

The design variables and cost of all the beams of Example 4.3 are tabulated in Table 4.2.

TABLE 4.2 : DESIGN VARIABLES AND COST OF BEAMS FOR EXAMPLE 4.3

Variable	Deterministic Design		Probabilistic Design					
	unexposed	exposed	Unexposed		Probability of failure			
			10 ⁻⁵	10 ⁻⁴	10 ⁻⁵	10 ⁻⁶	10 ⁻⁷	10 ⁻⁸
b (mm)	325	350	320	325	330	335	345	350
D (mm)	540	570	530	535	550	560	570	575
A _{st} (mm ²)	1256	1457	1256	1256	1335	1395	1413	1457
f _{ck} (N/mm ²)	20	20	20	20	20	20	20	20
r _f	-	0.71	-	0.71	0.71	0.71	0.71	0.71
f _y (N/mm ²)	415	415	415	415	415	415	415	415
M _e (kNm)	120	120	120	120	120	120	120	120
M _r /M _e	1.5	1.5	1.76	1.69	1.81	1.92	2.035	2.15
C _{Me}	-	-	0.15	0.15	0.15	0.15	0.150	0.15
C _{Mr}	-	-	0.053	0.063	0.063	0.063	0.063	0.063
Cost/m (Rs.)	225	251	221	224	234	240	247	253

4.7 Results and Discussion

The ratio of moment capacity of an exposed beam to unexposed beam can be obtained using Figs. 4.1 to 4.3, if the percentage of steel, acid consumed by concrete and yield strength of steel are specified. It is found that at 1.8, 1.0, 0.3 percentage reinforcement for f_y 250 , 415 and 500 grades, both sections become over-reinforced and hence no improvement in moment capacity. It can be seen from the Figs. 4.1 to 4.3 that there is a distinct change in $M_r(V)/M_r$, a point where the exposed beam changes from under-reinforced to over-reinforced, whereas the unexposed one is still under-reinforced. The range of V considered is 0.0 to 2.5 l/m^2 . As long as both are under-reinforced the effect of acid on moment capacity is less when compared to a situation in which one beam is over-reinforced and the other is under-reinforced. When both are over-reinforced the ratio is proportional to r_f . A typical curve showing the variation of moment capacity with reduction in concrete strength is shown in Fig. 4.4. The rate of decrease in moment capacity of under-reinforced section is less compared to that of over-reinforced sections. The rate of decrease of strength increases with increase in percentage of tension reinforcement. The rate of decrease in over-reinforced sections is proportional to the strength reduction factor.

For example if the reduction in strength of concrete is 10%, the reduction in moment capacity is 1.1%, 1.5% and 2.4% for $p_t = 0.4$, 0.6 and 0.8 respectively. Similarly, for a strength reduction of 60% of concrete the moment capacity is same for all the above three reinforcement ratios, because they become over-reinforced sections.

Figure 4.5 can be used to find the minimum cover required for a given exposure condition and life of a member. Otherway, the expected life of a member of a structure based on cover thickness criterion can also be estimated. For example: if t_p is known, V can be obtained and then by using Fig. 3.13 and Eq.(3.5) the time t in years can be obtained for the given conditions. This time is nothing but the time required for the acid to penetrate to a depth, t_p . So this can be taken as a life of the structure. If the acid penetrates to the level of steel, corrosion is initiated and the deterioration of the member accelerates due to acid attack as well as swelling attack caused by rusting of steel.

In the deterministic design, if the required moment of resistance and the strength of concrete and steel are known, the member can be proportioned using Eqs. (4.5) and (4.12). For unexposed beams Eqs. (4.1) to (4.4) or any standard design tables can be used. Using appropriate

equation from (4.18) to (4.24) the coefficient of variation of M_r can be calculated, if the statistical variation in design parameters is specified. After obtaining it, the ratio M_r/M_e can be interpolated using Figs. 4.6 to 4.12 for the coefficient of variation of external moment and probability of failure. These cover probability of failure in the range of 10^{-3} to 0.5, C_{Me} 0.0 to 0.175 and C_{Mr} 0.0 to 0.150. It can be seen that, if the variability of moment capacity and external moment is more, it is not possible to design for higher reliability. For example, if M_r/M_e is 3, $C_{Me} = 0.1$ and $C_{Mr} = 0.15$ the member cannot be designed for probability of failure less than $0.5(10^{-5})$ (Fig.4.9). Similarly for $M_r/M_e = 1.5$, probability of failure = 10^{-5} , the possible combinations of C_{Me} and C_{Mr} are: 0.0, 0.073; 0.05, 0.07; 0.075, 0.06; 0.1, 0.041; 0.117, 0.0. This can also be verified using the Eqs. (4.29) to (4.33).

Although standard design tables are available for proportioning the member, design charts for $f_{ck} = 20 \text{ N/mm}^2$ and $f_y = 415 \text{ N/mm}^2$ are included for quick reference. The results of Example 4.2 are tabulated in Table 4.1. Example 4.2 is a design of an exposed and unexposed beam by deterministic approach. The strength of concrete is reduced by 57 %, whereas the cost is increased by 8.3% (compared with an unexposed one). Both cross-sections are shown in

Fig. 4.14. A design example (Example 4.3) is taken up to study the variation of cost of beam designed by probabilistic approach. In this, both exposed and unexposed ones are studied. Deterministic design is also included for comparison purpose. The size of cross-section for various cases are shown in Figs. 4.15a - 4.15h. Figure 4.15a and b are the cross sections of unexposed and exposed ones designed by deterministic approach. The reduction in strength is 29% whereas the increase in cost is 10%. Figure 4.15c is the cross section of an unexposed beam designed by probabilistic approach for a probability of failure 10^{-5} . The cost of an unexposed beam by both methods has turned out to be almost same. Figures 4.15d to 4.15h are cross sections of exposed beams designed by probabilistic approach for probability of failure 10^{-4} , 10^{-5} , 10^{-6} , 10^{-7} and 10^{-8} respectively. From Fig. 4.16, it can be seen that the cost of an exposed beam increases with increase in reliability. But the rate at which it increases is more in higher probabilities of failure and decreases while approaching towards lower ones. The cost of an exposed beam designed by probabilistic approach is compared with the unexposed one designed by deterministic approach. The increase in cost is 0.0%, 4%, 6.7%, 9.8% and 12.5% for probability of failure 10^{-4} , 10^{-5} , 10^{-6} , 10^{-7} and 10^{-8} respectively.

$f_{ck} = 20 \text{ N/mm}^2$

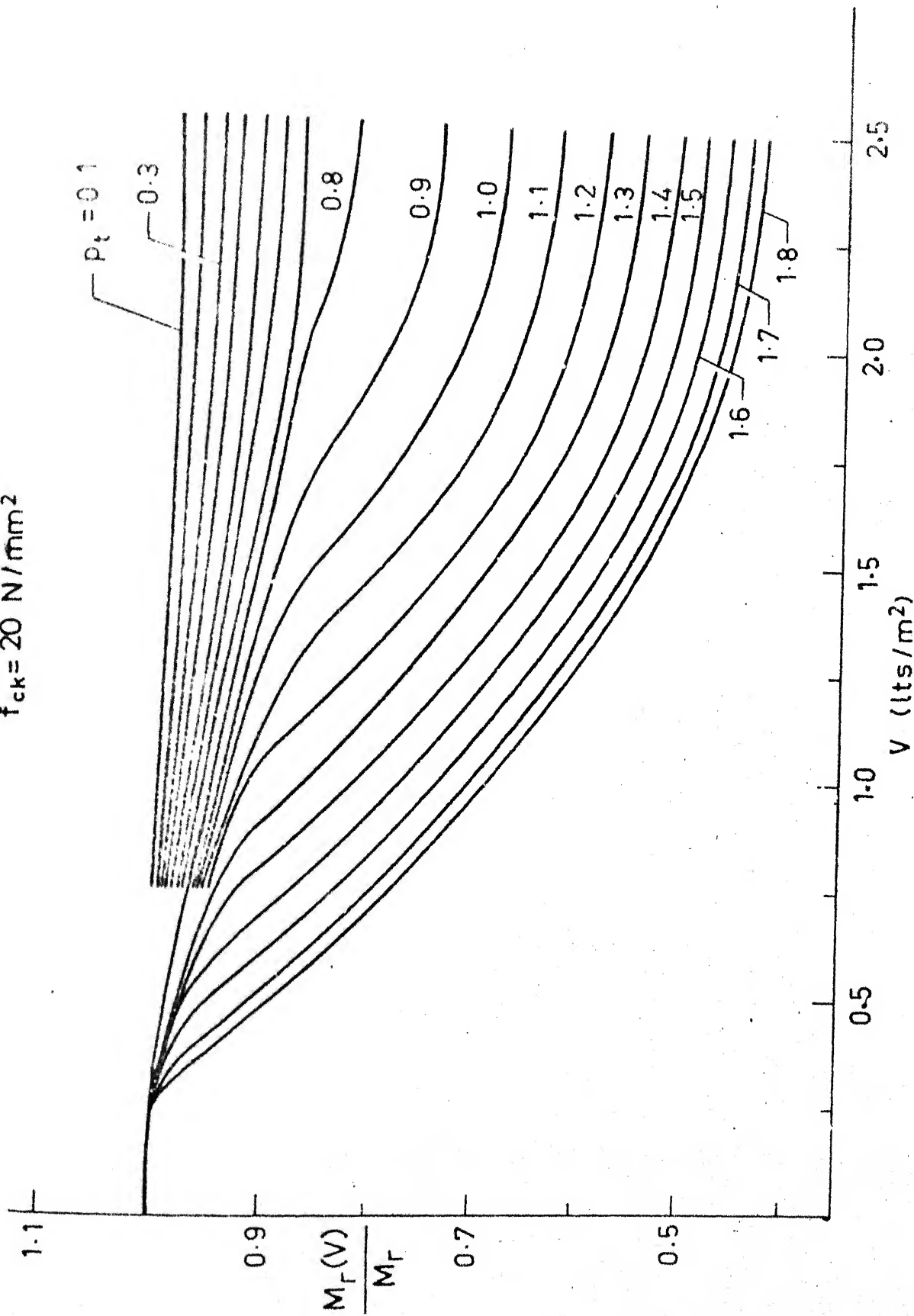


FIG. 4.1 VARIATION OF $M_r(V)/M_r$ WITH ACID CONSUMED ($f_y = 250 \text{ N/mm}^2$)

$f_{ck} = 20 \text{ N/mm}^2$

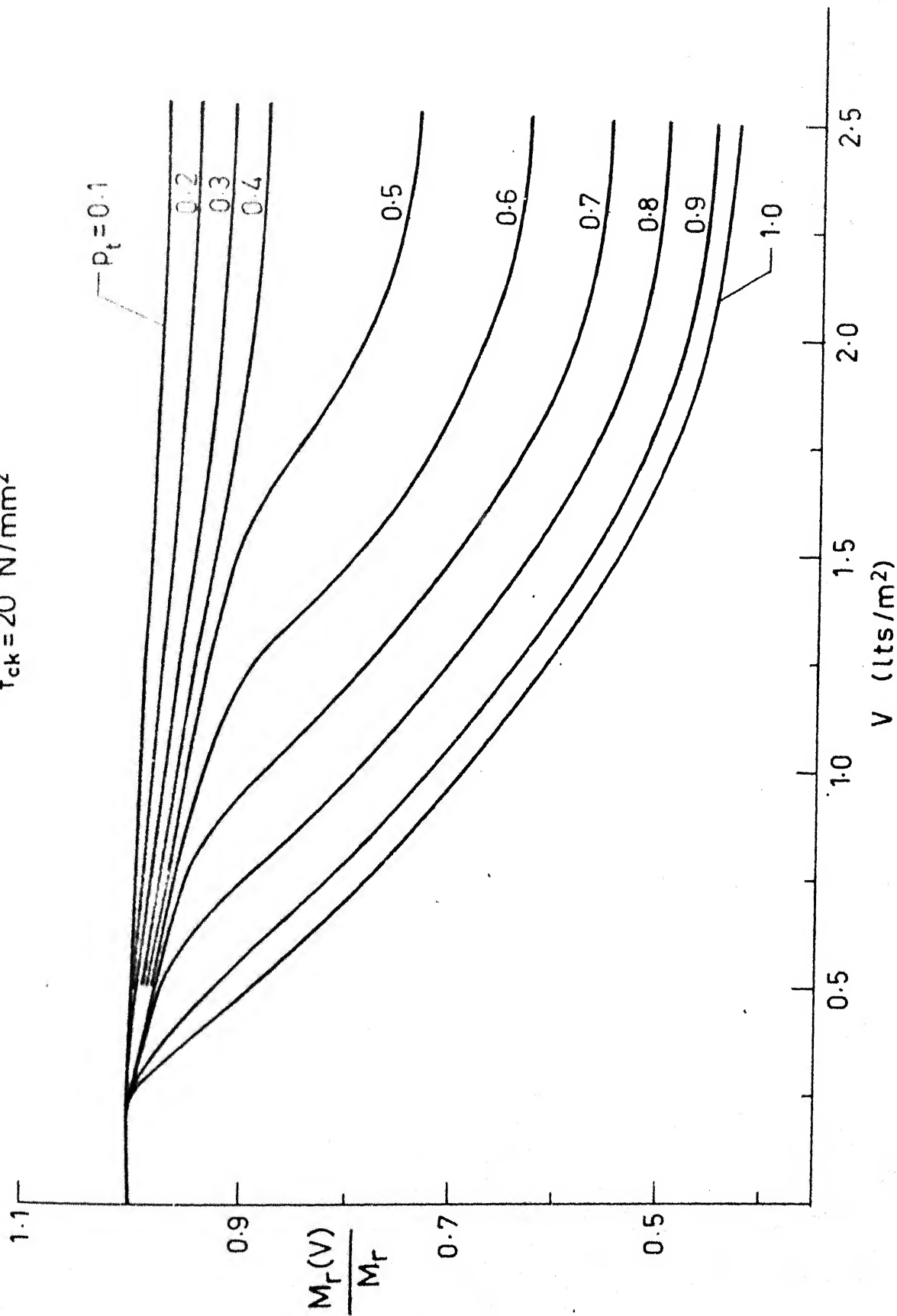


FIG. 4.2 VARIATION OF $M_r(V)/M_r$ WITH ACID CONSUMED ($f_y = 415 \text{ N/mm}^2$)

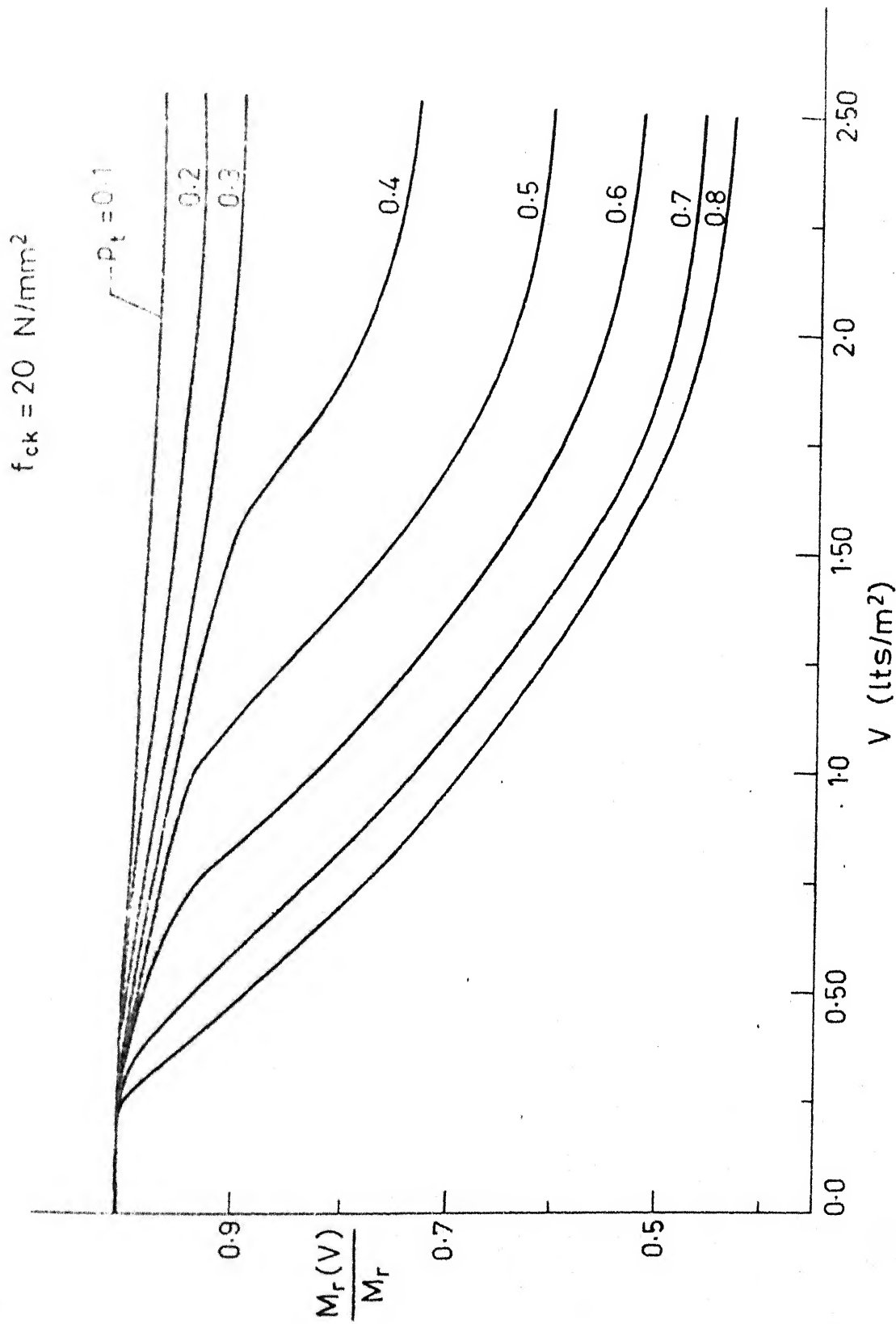


FIG. 4.3 VARIATION OF $M_r(V)/M_r$ WITH ACID CONSUMED ($f_y = 500 \text{ N/mm}^2$)

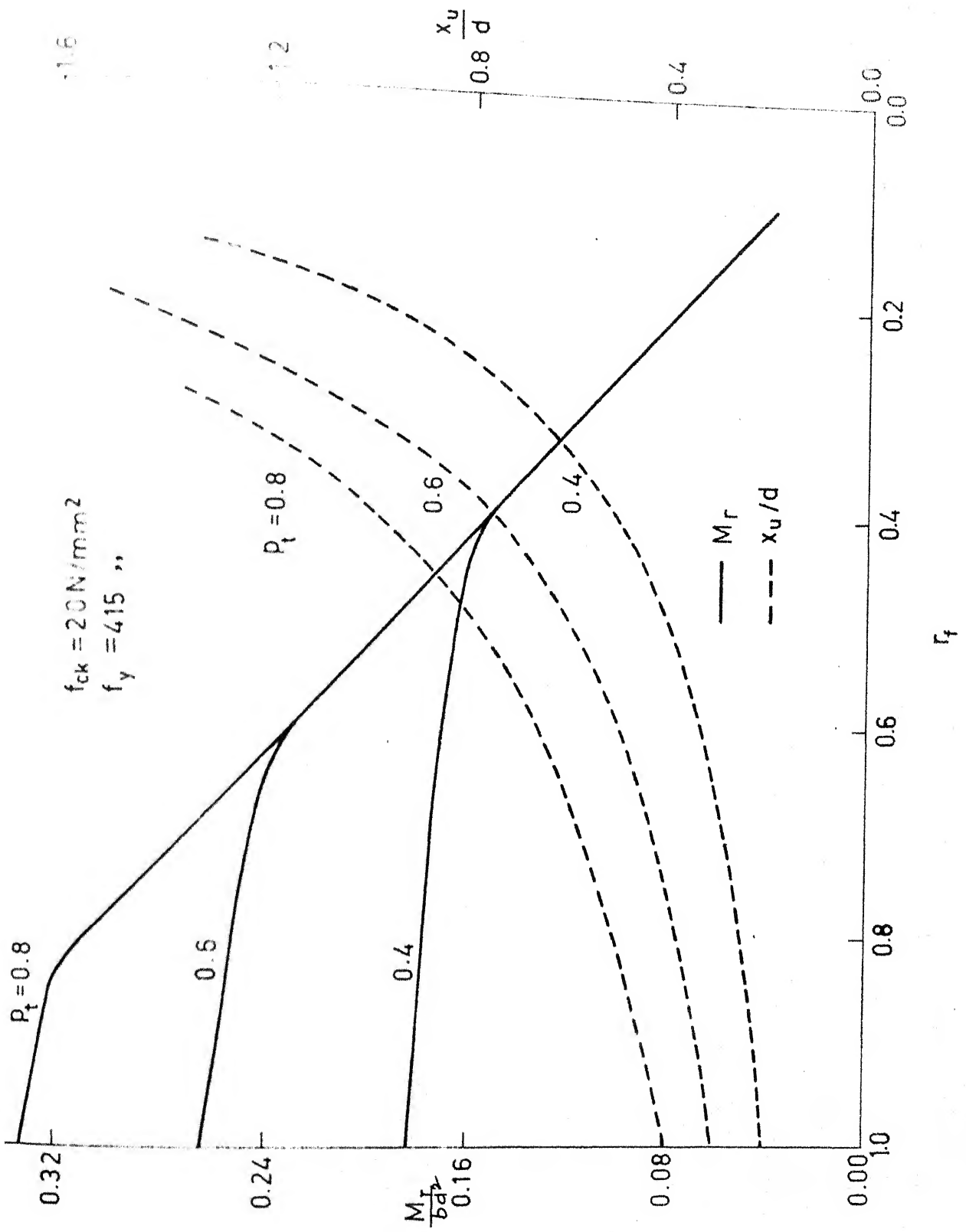


FIG. 4.4 VARIATION OF MOMENT CAPACITY WITH STRENGTH REDUCTION

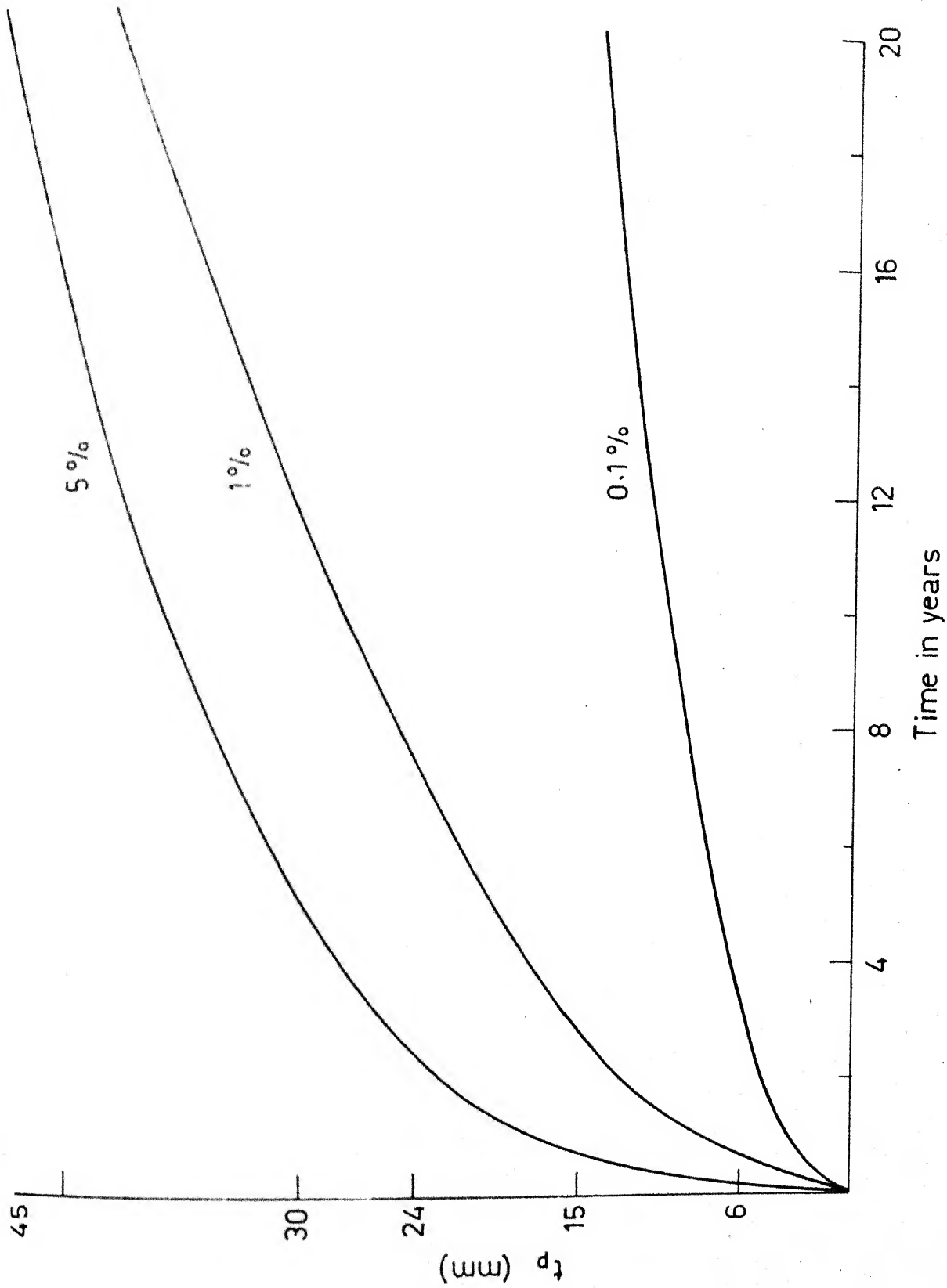


FIG. 4.5 VARIATION OF DEPTH OF PENETRATION OF ACID WITH TIME

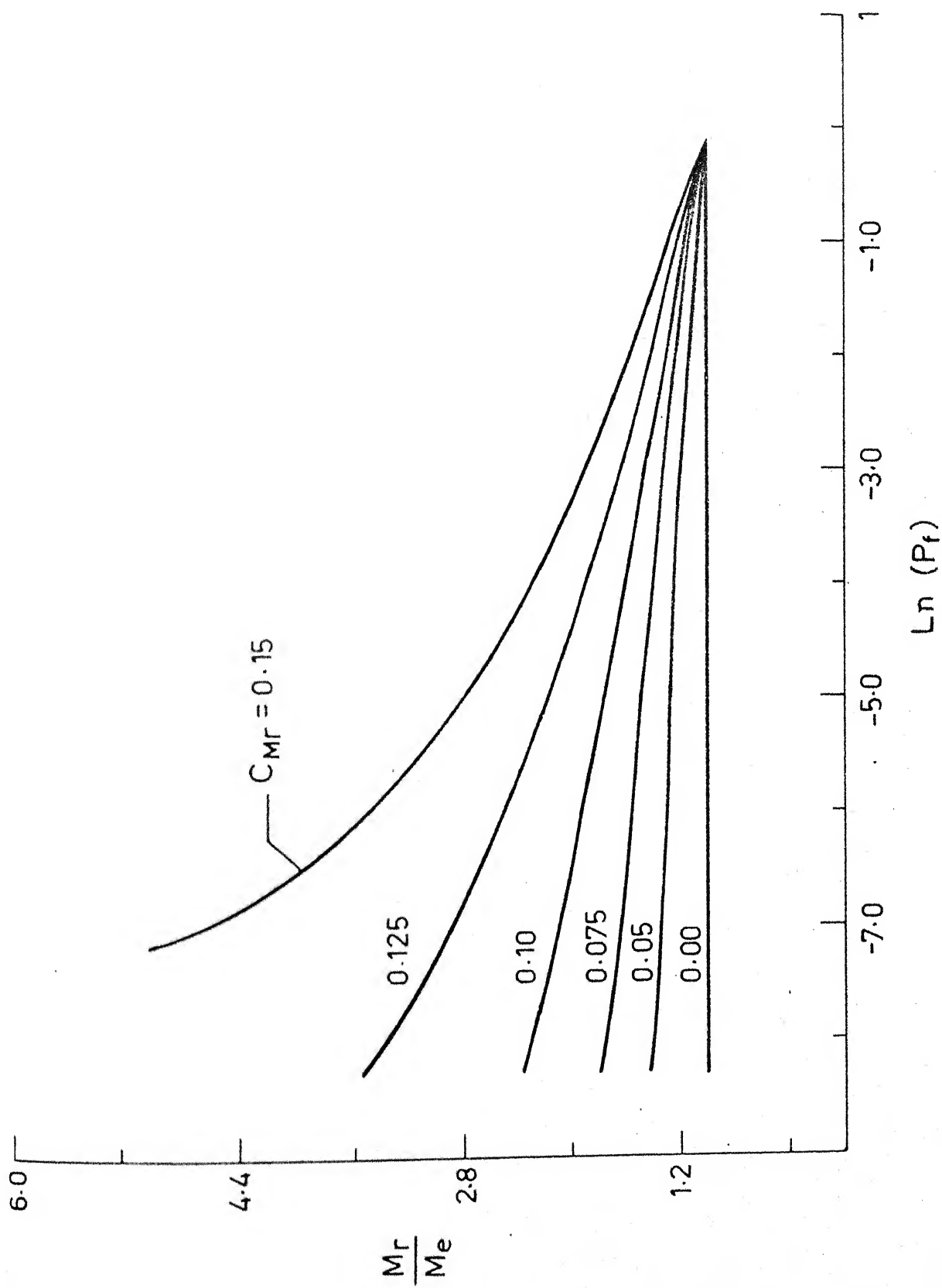


FIG. 4.6 VARIATION OF M_r / M_e WITH PROBABILITY OF FAILURE ($C_{Me}=0.0$)

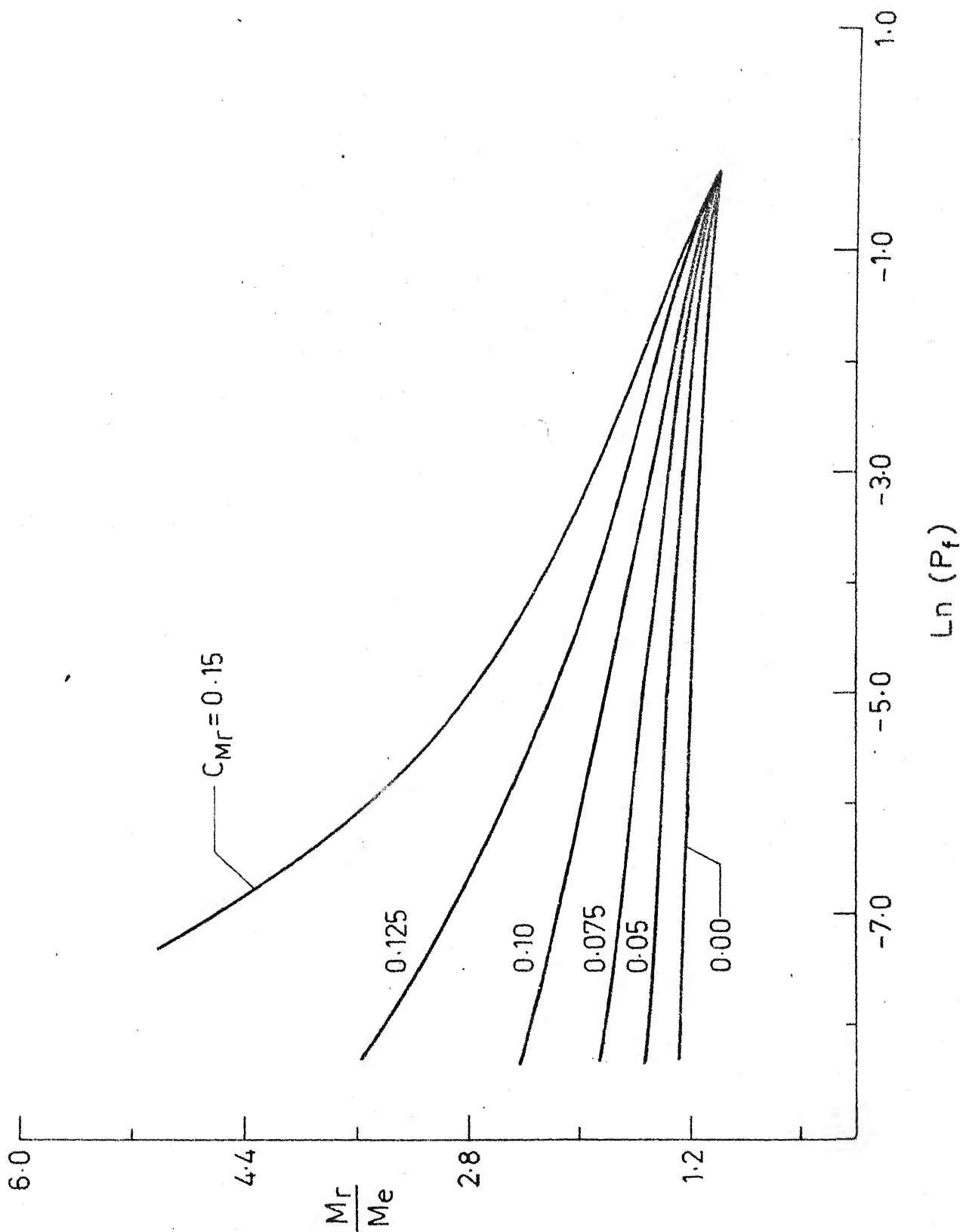


FIG. 4.7 VARIATION OF M_r / M_e WITH PROBABILITY OF FAILURE ($C_{Me} = 0.05$)

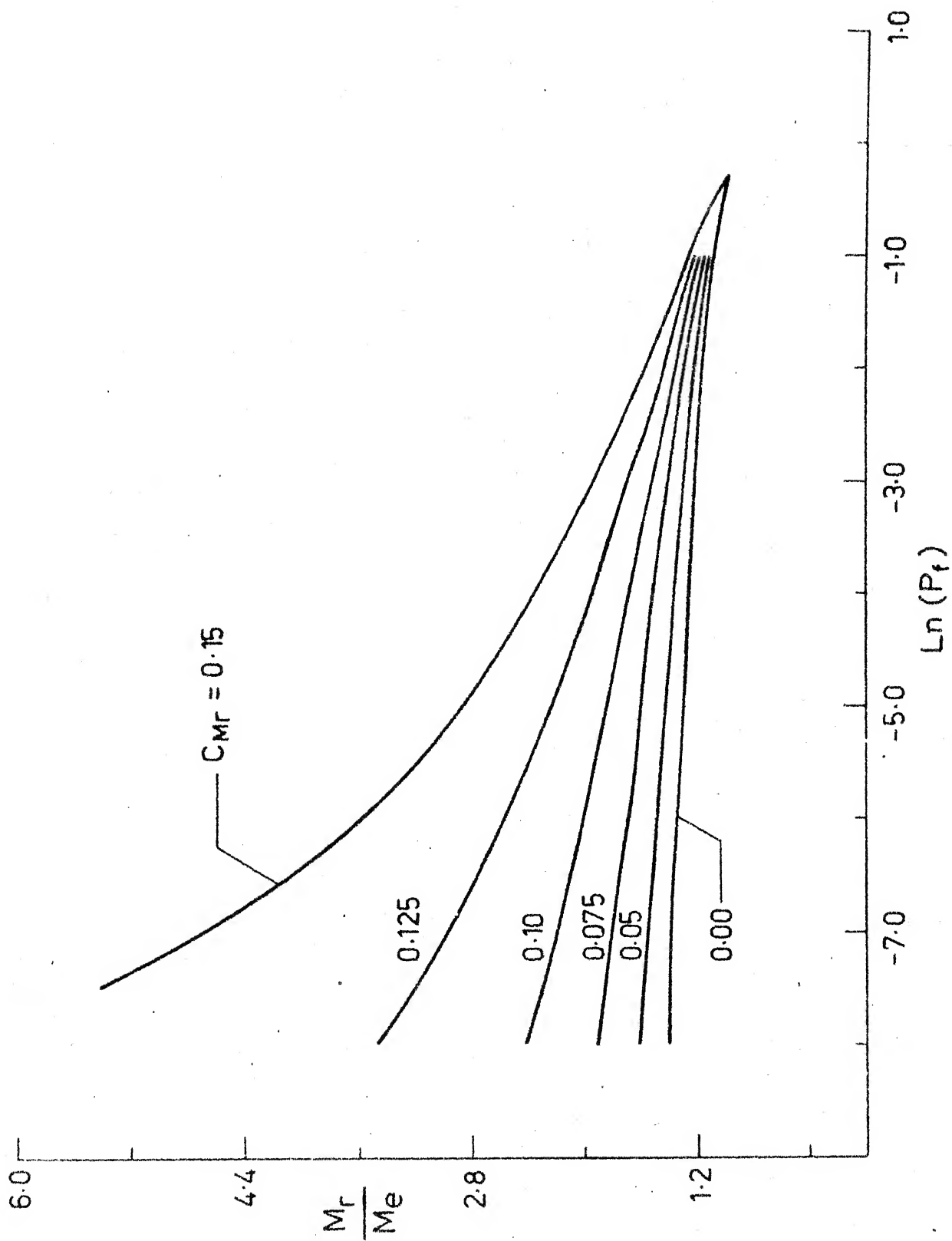


FIG. 4.8 VARIATION OF M_r/M_e WITH PROBABILITY OF FAILURE ($C_{Me} = 0.075$)

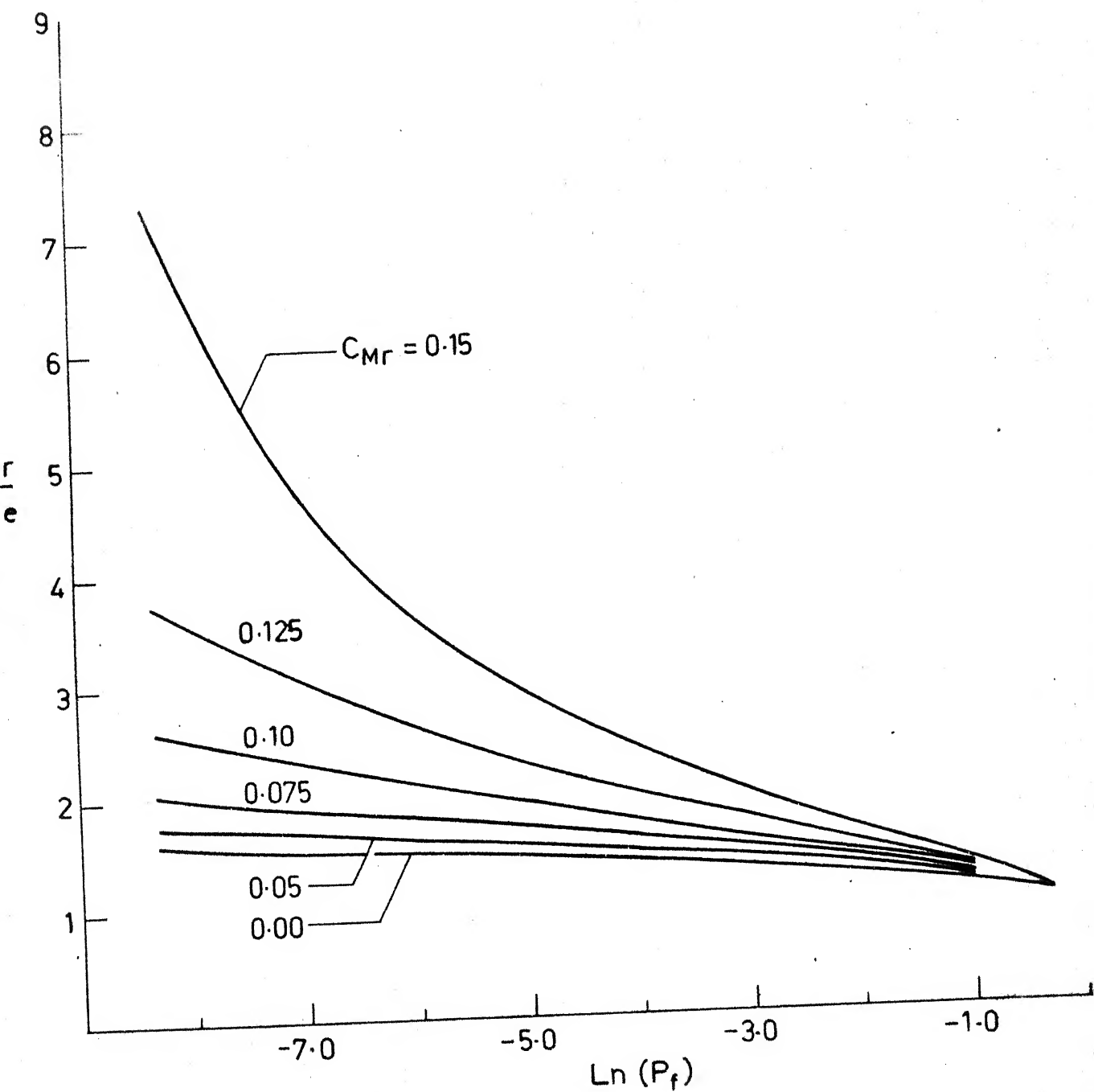


FIG. 4.9 VARIATION OF M_r/M_e WITH PROBABILITY OF FAILURE
($C_{Me}=0.10$)

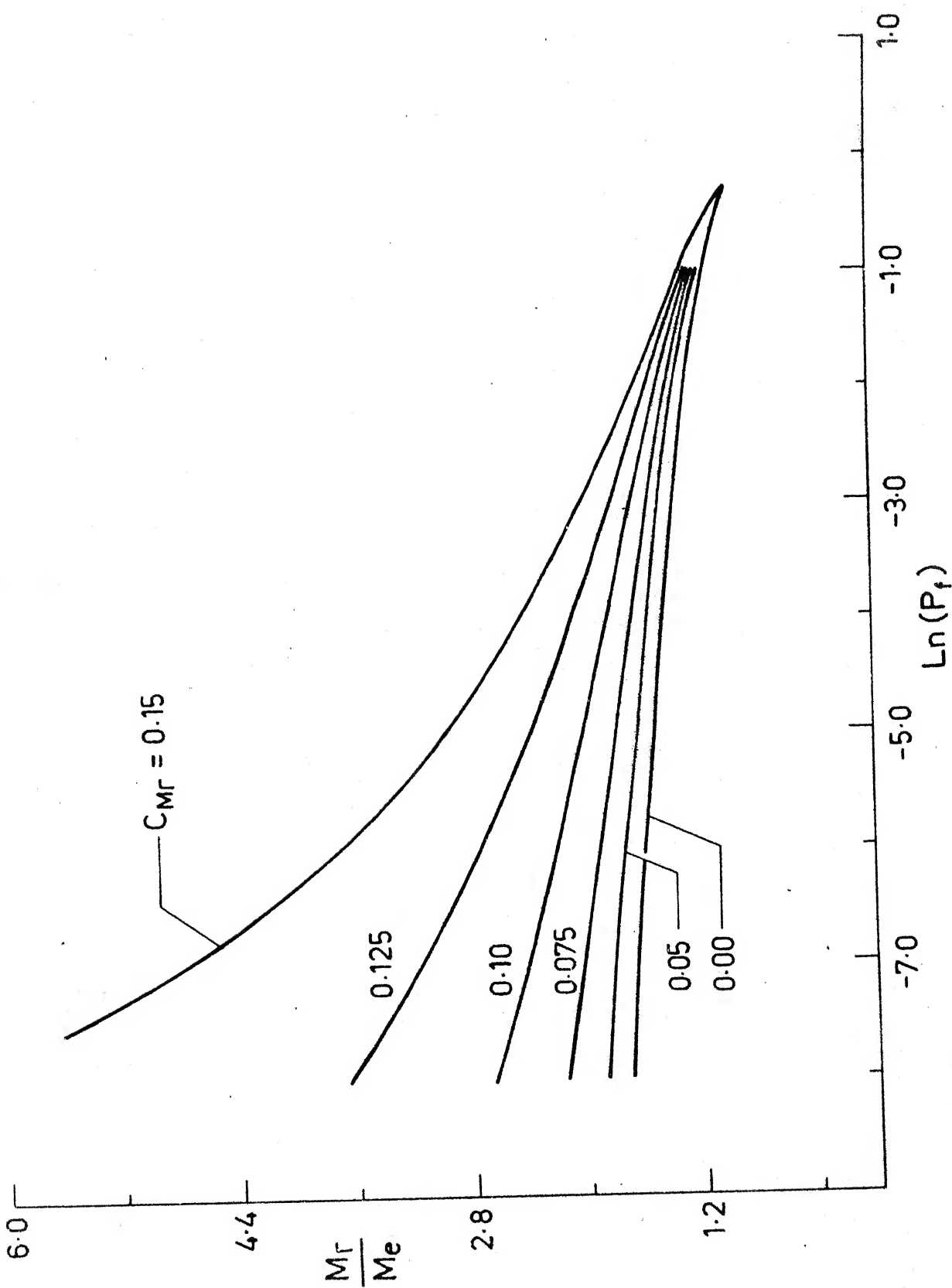


FIG. 4.10 VARIATION OF M_r / M_e WITH PROBABILITY OF FAILURE ($C_{Me} = 0.125$)

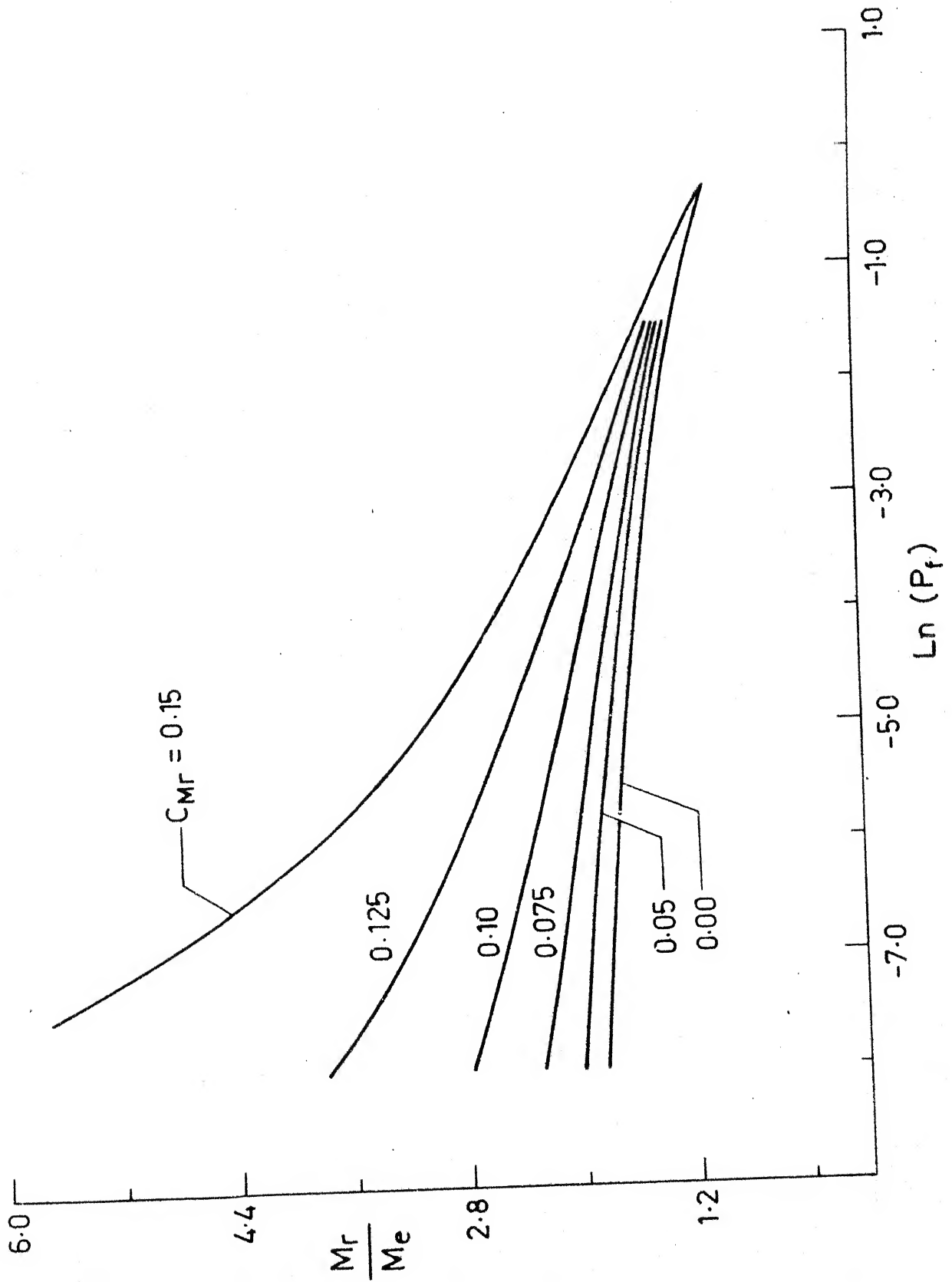


FIG. 4.11 VARIATION OF M_r/M_e WITH PROBABILITY OF FAILURE ($C_{Me}=0.15$)

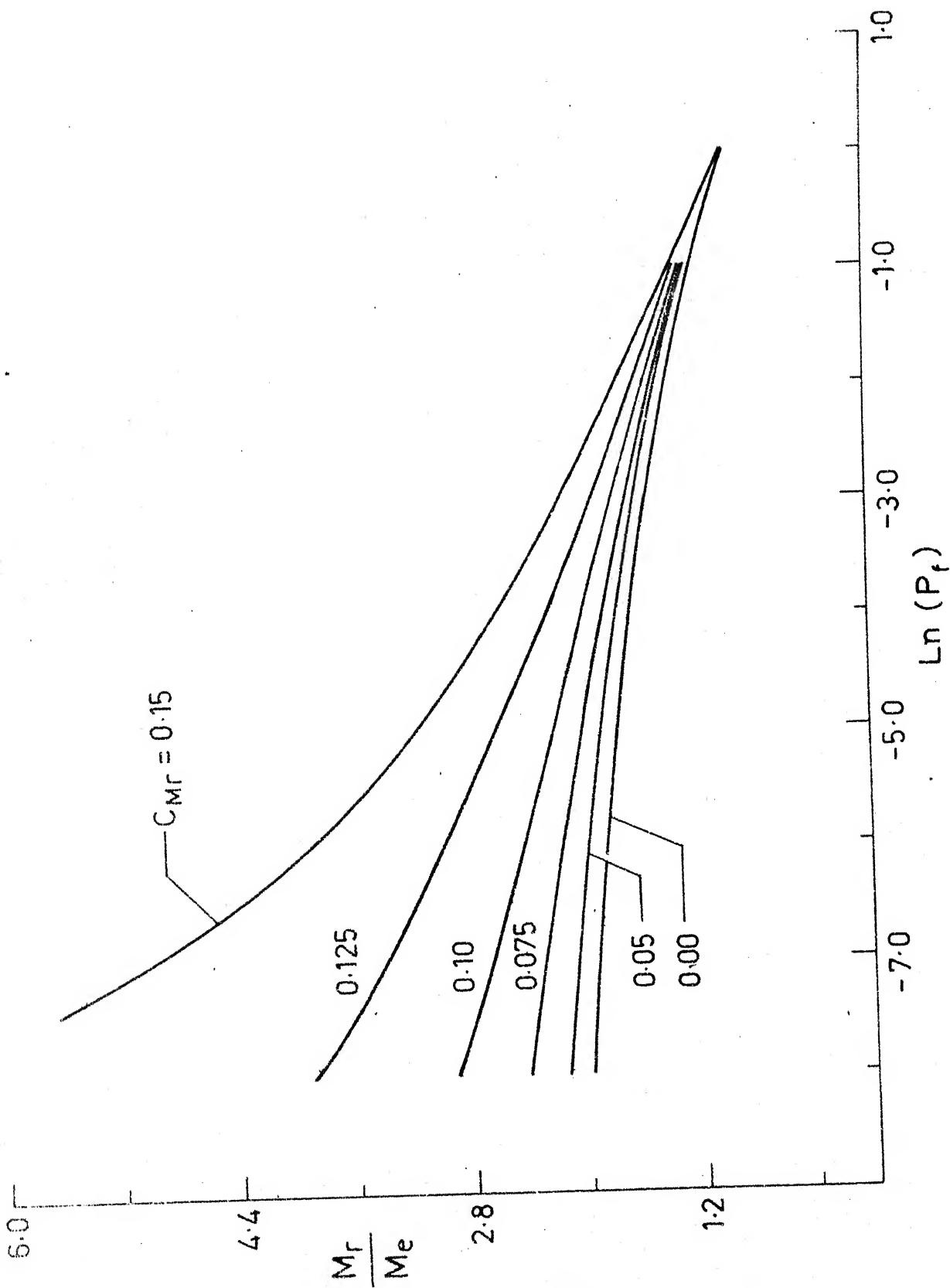


FIG. 4.12 VARIATION OF M_r/M_e WITH PROBABILITY OF FAILURE ($C_{Me} = 0.175$)

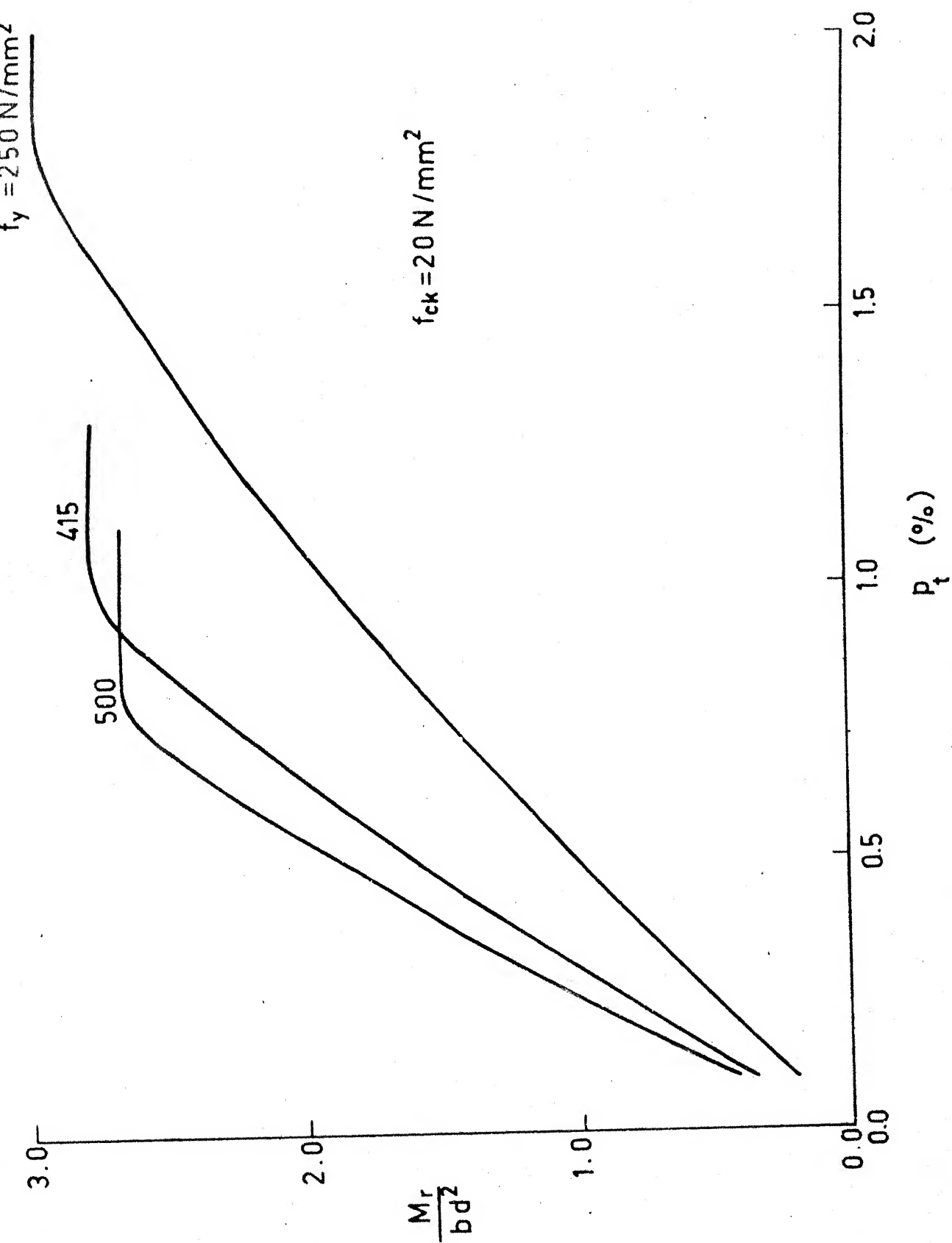
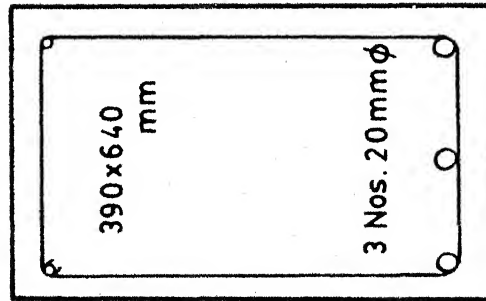
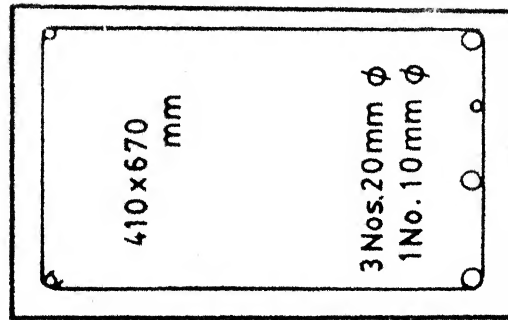


FIG. 4.13 DESIGN CHART FOR RECTANGULAR BEAM (LSD)



(a)



(b)

FIG. 4.14 BEAM CROSS SECTIONS FOR EXAMPLE 4.2

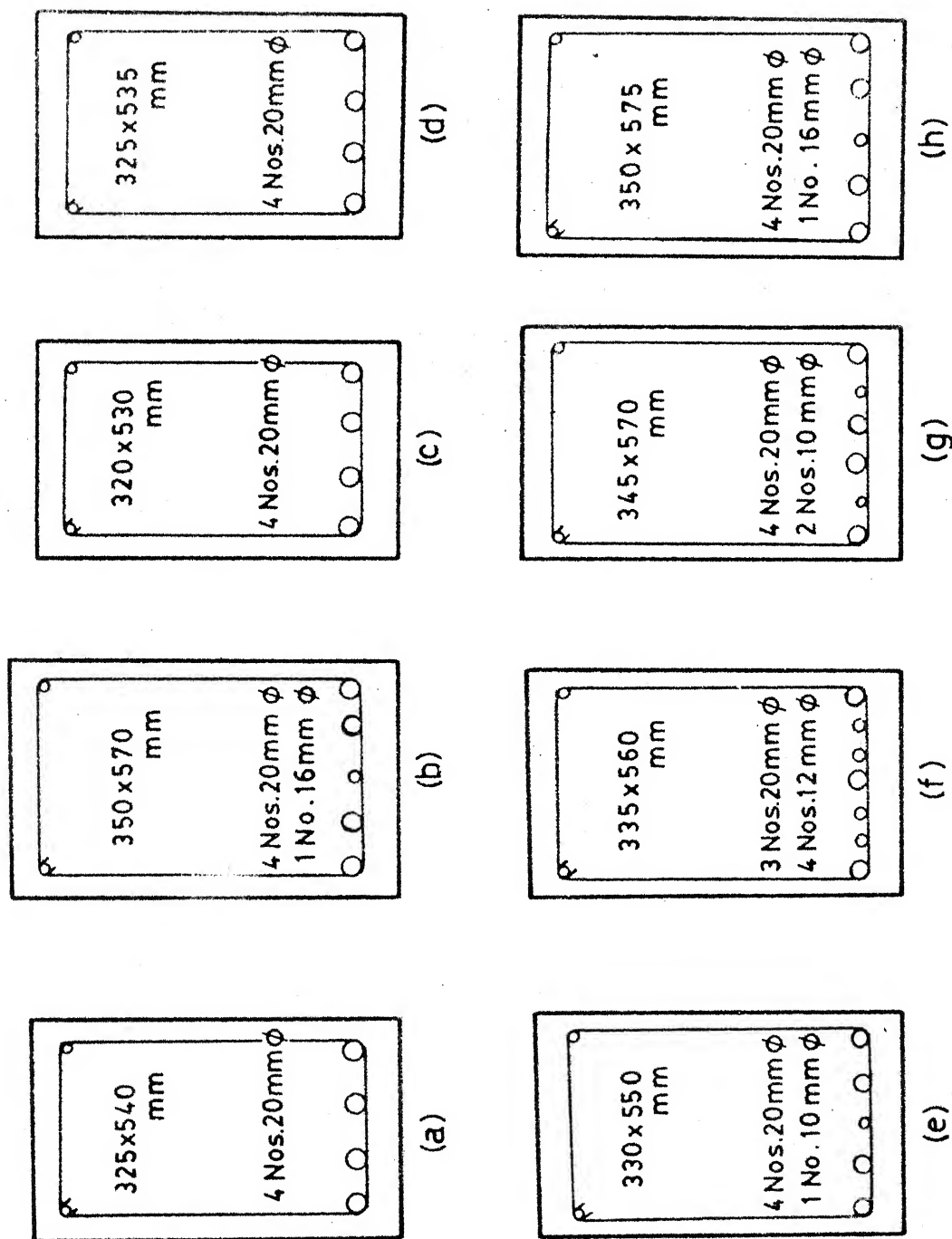


FIG. 4.15 BEAM CROSS SECTIONS FOR EXAMPLE 4.3

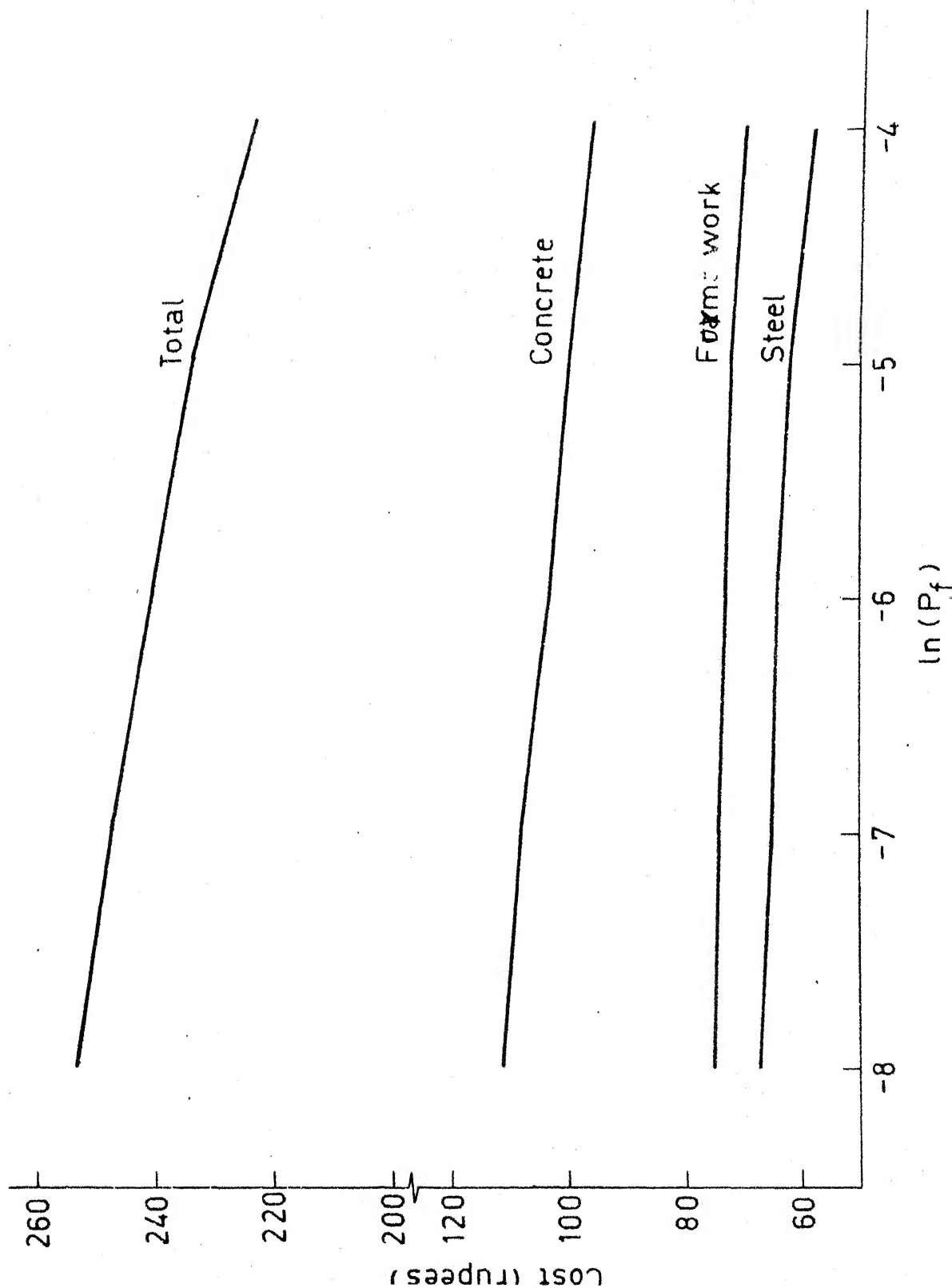


FIG. 4.16 VARIATION OF COST OF BEAM WITH PROBABILITY OF FAILURE

CHAPTER V

PROBABILITY OF FAILURE AND HAZARD RATE OF A DETERIORATING MEMBER

5.1 Introduction

In engineering structures uncertainties invariably occur through either strength or loading or both. The structures are to withstand vagaries of nature also. Certain loads can be estimated deterministically within a reasonable degree of accuracy. But most of the loads are random in nature. Similarly, steel manufactured under controlled environment has low coefficient of variation compared to concrete. So the data on strength and forces are required to establish the variabilities. The observed data on a particular type of structures are useful in establishing the appropriate behaviour of those structures. This data can also be used in decision analysis. Procedure for finding out the probability of failure, probability density function and hazard rate of a deteriorating member is presented. Examples are worked out to explain the use of expressions developed, and to incorporate the known data in determining the probability of failure. The coefficient of variation of 0.1 for external moment and 0.05 in characteristic yield strength of steel

are used. The coefficient of variation of concrete strength is calculated for each case. The results are tabulated in Tables 5.1 - 5.4. Reliability, cumulative probability of failure and hazard rate of a deteriorating member are calculated.

5.2 Estimation of Probability of Failure

Let $f_{Me}(m_e)$ and $f_{Mr}(m_r)$ be the probability density functions of external moment and moment of resistance respectively. These two distributions are assumed as Gaussian distributions [110].

$$\begin{aligned} \text{Probability of failure} = p &= P(M_r < M_e) \\ &\text{or } P(M_e - M_r < 0) \end{aligned} \quad (5.1)$$

The overlapping area between the two functions shown in Fig. 5.1a is nothing but the probability of failure p . The probability of an external moment lying in a small interval dM_e is (Fig. 5.1b)

$$P\left(M_{eo} - \frac{dM_e}{2} \leq m_e \leq M_{eo} + \frac{dM_e}{2}\right) = f_{Me}(m_{eo}) dM_e \quad (5.2)$$

The probability that the resisting moment less than a certain external moment M_{eo} is

$$P(M_r < M_{eo}) = \int_{-\infty}^{M_{eo}} f_{Mr}(m_r) dM_r \quad (5.3)$$

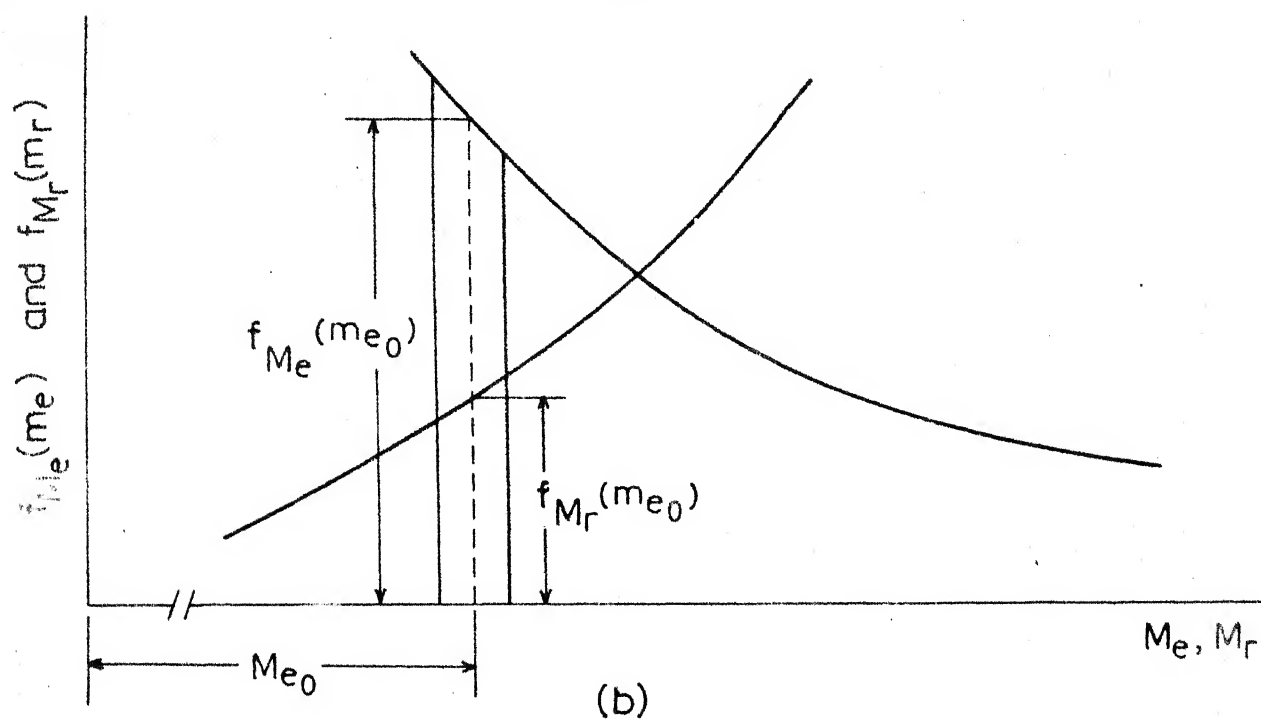
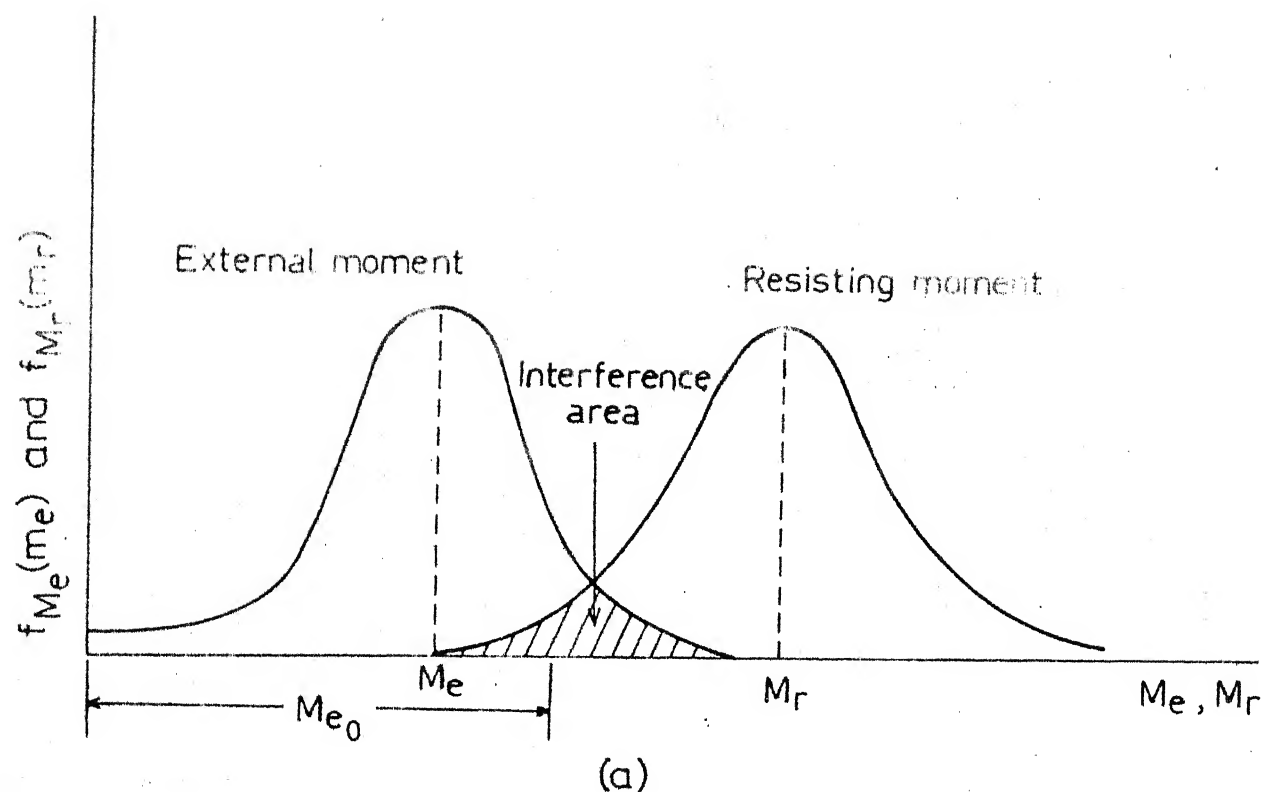


FIG. 5.1 (a) INTERFERENCE OF EXTERNAL MOMENT AND MOMENT OF RESISTANCE
(b) ENLARGED DIAGRAM OF INTERFERENCE AREA

The probability for the external moment lying in the interval dM_e and the moment of resistance not exceeding the external moment given by the small interval dM_e is

$$f_{Me}(m_{eo}) dM_e \left[\int_{-\infty}^{M_{eo}} f_{Mr}(m_r) dM_r \right] \quad (5.4)$$

(Because external moment and moment of resistance are independent

Now, the probability of failure of the beam is the probability that the resisting moment M_r less than the external moment M_e for all possible values of M_e is given by

$$\begin{aligned} P &= \int_{-\infty}^{\infty} f_{Me}(m_e) \left[\int_{-\infty}^{M_{eo}} f_{Mr}(m_r) dM_r \right] dM_e \\ &= \int_{-\infty}^{\infty} F_{Mr}(M_{eo}) f_{Me}(m_e) dM_e \end{aligned} \quad (5.5)$$

Alternatively, one can calculate the probability of failure on the basis that the external moment is more than the resisting moment. The probability that the moment of resistance M_r within a small interval dM_r is

$$P(M_{ro} - \frac{dM_r}{2} \leq m_r \leq M_{ro} + \frac{dM_r}{2}) = f_{Mr}(m_{ro}) dM_r \quad (5.6)$$

The probability that the external moment greater than M_{ro} is given by

$$P(M_e \geq M_{ro}) = \int_{M_{ro}}^{\infty} f_{Me}(m_e) dM_e \quad (5.7)$$

The probability of M_r lying in the small interval dM_r and external moment exceeding M_{ro} is

$$f_{Mr}(M_{ro}) dM_r \left[\int_{M_{ro}}^{\infty} f_{Me}(m_e) dM_e \right] \quad (5.8)$$

Therefore the probability of failure of the beam for all possible values of M_r is

$$\begin{aligned} p &= \int_{-\infty}^{\infty} f_{Mr}(m_r) \left[\int_{M_{ro}}^{\infty} f_{Me}(m_e) dM_e \right] dM_r \\ &= \int_{-\infty}^{\infty} F_{Me}(M_{ro}) f_{Mr}(m_r) dM_r \end{aligned} \quad (5.9)$$

The other way of finding the probability of failure is by considering a new random variable function of M_e and M_r .

$$m = m_e - m_r \quad (5.10)$$

m_e and m_r are random variables, so their mean is also a random variable with

$$\begin{aligned} \text{mean} &= M = M_e - M_r \quad \text{and} \\ \text{variance} \quad s_M^2 &= s_{Me}^2 + s_{Mr}^2 \end{aligned}$$

$$\text{Probability of failure of a beam} = P(M > 0) \quad (5.11)$$

$$\begin{aligned} [107] \cdot \quad f_M(m) &= \frac{1}{\sqrt{2\pi} \cdot s_{Me} \cdot \sqrt{2\pi} \cdot s_{Mr}} \int_{-\infty}^{\infty} \exp \left[-\frac{1}{2} \left(\frac{m+m_r-M_e}{s_{Me}} \right)^2 \right. \\ &\quad \left. + \left(\frac{m_r-M_r}{s_{Mr}} \right)^2 \right] dM_r \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2\pi s_{Me} s_{Mr}} \exp \left[-\frac{1}{2} \left(\frac{M_r}{s_{Mr}} \right)^2 + \left(\frac{m - M_e}{s_{Me}} \right)^2 \right] \\
&\int_{-\infty}^{\infty} \exp \left[-\frac{1}{2} \frac{m_r^2 (s_{Me}^2 + s_{Mr}^2)}{s_{Me}^2 \cdot s_{Mr}^2} - 2M_r \left(\frac{M_r}{s_{Mr}^2} + \frac{m - M_e}{s_{Me}^2} \right) \right]^2 dM_r
\end{aligned}
\tag{5.12}$$

and this will further simplify to

$$f_M(m) = \frac{1}{\sqrt{2\pi(s_{Me}^2 + s_{Mr}^2)}} \exp \left[-\frac{1}{2} \left\{ \frac{m - (M_e - M_r)}{\sqrt{s_{Me}^2 + s_{Mr}^2}} \right\}^2 \right]
\tag{5.13}$$

Let p_i be the probability that the beam has failed in the year i given that it has not failed prior to i th year and E_i be an event of the failure of beam in year i [111]

$$\therefore P[E_i] = p_i \text{ and } P[\bar{E}] = 1 - p_i
\tag{5.14}$$

where bar over the variable indicates the complement of that event.

Probability of survival of the beam in first year is

$$P[\bar{E}_1] = 1 - p_1
\tag{5.15}$$

Probability that the beam does not fail in either of first two years is

$$\begin{aligned}
P[\bar{E}_1 \cap \bar{E}_2] &= P[\bar{E}_1] P[\bar{E}_2 | \bar{E}_1] \\
&= (1-p_1)(1-p_2)
\end{aligned}
\tag{5.16}$$

(Because E_1 and E_2 are independent events).

$$\begin{aligned} \therefore P [\text{survival of beam through } j \text{ years}] &= R(j) \\ &= P [\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3 \dots \dots \cap \bar{E}_j] \end{aligned} \quad (5.17)$$

By using simple extension of Eqs. (5.15) and (5.16),
Eq. (5.17) becomes

$$R(j) = \prod_{i=1}^j (1 - p_i) \quad (5.18)$$

$$\begin{aligned} P [\text{failure of beam in } j \text{ years}] &= p_j \\ &= P [E_j \mid \text{survival upto } (j-1) \text{ years}] \\ &= P [E_j \mid \bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3 \dots \dots \cap \bar{E}_{j-1}] \end{aligned} \quad (5.19)$$

$$\begin{aligned} \therefore P [\text{failure of beam in } j \text{th year for the first time}] &= f(j) \\ &= P [\text{no survival in } j \mid \text{previous survival}] \\ &\quad P [\text{previous survival}] \\ &= p_j \prod_{i=1}^{j-1} (1 - p_i) \end{aligned} \quad (5.20)$$

Suppose the beam has survived (probability of failure is zero) in the first year means $p_1 = 0$ which also implied $f(1) = 0$. Similarly, if it has survived in first two years $p_1 = 0$; $p_2 = 0$, therefore $f(2) = 0$. By simply extending, equation (5.20) can be modified as

$$f(j) = p_j \prod_{i=Ns+1}^{j-1} (1 - p_i) \quad (5.21)$$

where N_s - number of years survived.

Hazard rate $H(i)$ is defined as the ratio of number of failures during a particular unit interval to the average population during that interval [110]

For discrete case

$$H(i) = \frac{2 p_i}{R(i-1) + R(i)} \quad (5.22)$$

For continuous case

$$H(i) = \frac{f(i)}{R(i)} \quad (5.23)$$

5.3 Examples

Example 5.1

Design a reinforced concrete rectangular beam, simply supported on both edges with a span of 6 m, to carry a live load of 20 kN/m. Use concrete M20 grade and $f_y = 415 \text{ N/mm}^2$. Calculate the probability of failure of the beam for $C_{Me} = 0.1$.

$$\text{Live load} = 20 \text{ kN/m}$$

$$\text{Dead load} = 5 \text{ kN/m (inclusive of all)}$$

$$\text{Total load} = 25 \text{ kN/m}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$\text{Design load} = 25 \times 1.5 = 37.5 \text{ kN/m}$$

$$\therefore \text{Maximum bending moment} = \frac{37.5 \times 6 \times 6}{8} = 168.75 \text{ kN m}$$

$$k_b = 0.48 \text{ for } f_y = 415 \text{ N/mm}^2 [109]$$

$$\begin{aligned} M_r &= 0.36 f_{ck} b d^2 k_b (1 - 0.42 k_b) \\ &= 1.8395 d^3 \text{ for } b = 2/3 d \end{aligned}$$

$$d = \sqrt[3]{(168.75 \times 10^6) / 1.8395} = 451 \text{ mm} \text{ Say } 455 \text{ mm}$$

$$b = \frac{2}{3} \times 451 = 300 \text{ mm}$$

$$\begin{aligned} A_{st} &= \frac{0.36 f_{ck} b d k_b}{0.87 f_y} \\ &= \frac{0.36 \times 0.48 \times 20 \times 300 \times 455}{415 \times 0.87} = 1295 \text{ mm}^2 \end{aligned}$$

Provide 4 nos. 20 mm ϕ bars and 1 no. 10 mm ϕ bar

$$\therefore \text{Total steel area provided} = 1335 \text{ mm}^2$$

Moment capacity of the section without partial safety factors (M_u) is computed here

$$b = 300 \text{ mm} ; d = 455 \text{ mm}$$

$$A_{st} = 1335 \text{ mm}^2$$

$$p_t = \frac{1335}{300 \times 455} \times 100 = 0.98$$

$$\frac{M_u}{b d^2} = 3.43 \text{ for } p_t = 0.98 \text{ from Fig. B.2}$$

$$\therefore M_u = 3.43 \times 300 \times 455^2 \times 10^{-6} = 212.5 \text{ kN m}$$

Calculation of coefficient of variation of M_r

$$\frac{x_u}{d} = \frac{415 \times 0.98}{54.27 \times 20} = 0.375 < k_b$$

Let $C_{fy} = 0.05$; $C_{fck} = 0.1$

$$C_{xud} = \sqrt{(0.05)^2 + (0.1)^2} = 0.112$$

$$C_{Mr} = \left[0.05^2 + \left(\frac{0.42 \times 0.375 \times 0.112}{1 - 0.42 \times 0.375} \right)^2 \right]^{\frac{1}{2}} = 0.055.$$

$$\frac{M_r}{M_e} = \frac{212.5}{112.5} = 1.89$$

$$Z = \frac{1 - \frac{M_r}{M_e}}{\sqrt{\left(C_{Me}^2 + \frac{M_r^2}{M_e^2} C_{Mr}^2\right)}} = \frac{1.0 - 1.89}{\sqrt{(0.1^2 + 0.055^2 \times 1.89^2)}} = -6.172$$

$$\text{Probability of failure} = \int_{-\infty}^{-6.172} e^{-x^2/2} dx = 0.354 \times 10^{-9}$$

Example 5.2 :

Design a reinforced concrete simply supported beam of 6 m span , exposed to 0.01 % concentration of sulphuric acid. The design load is 25 kN/m. The beam has to survive for 50 years. Use concrete grade M20 and $f_y = 415 \text{ N/mm}^2$. Calculate the probability of failure of a beam for $C_{Me} = 0.1$.

$$M_e = 168.75 \text{ kN m (from previous example)}$$

for 0.01 % concentration of sulphuric acid the constants are

$$B = 0.007 \quad \text{and} \quad C = 0.50 \quad \text{from Fig. 3.13 .}$$

$$t = 50 \text{ years}$$

$$V = 30 [1 - \exp(-0.007 \sqrt{50})] = 1.449$$

$$r_f = 1.14 - 0.5741 \times 1.449 - 0.1163 \times 1.449^2 = 0.552$$

$$k_b = 0.48 ; \quad b = 2/3 d$$

$$M_r = 0.36 \times 11.04 \times \frac{2}{3} d^3 \times 0.48(1 - 0.42 \times 0.48) = 1.015 d^3$$

$$d = \sqrt[3]{(168.75 \times 10^6 / 1.015)} = 550 \text{ mm}$$

$$b = 367 \text{ mm} \quad \text{say } 370 \text{ mm}$$

$$A_{st} = 0.198 \times 367 \times 550 \times \frac{11.04}{4.15} = 1063 \text{ mm}^2$$

Provide 3 nos. 20 mm ϕ and 2 nos. 10 mm ϕ bars.

$$\text{Total steel area provided} = 1100 \text{ mm}^2$$

Moment capacity (M_u) of the section is computed for the following properties.

$$b = 370 \text{ mm} ; \quad d = 550 \text{ mm} ; \quad A_{st} = 1100 \text{ mm}^2$$

$$p_t = \frac{1100 \times 100}{370 \times 550} = 0.54$$

$$\frac{M_u}{b d^2} = 2.055 \text{ for } p_t = 0.54 \quad \text{from Fig. B.2.}$$

$$M_u = 2.055 \times 370 \times 550^2 \times 10^{-6} = 229 \text{ kN m}$$

Calculation of C_{Mr}

$$\frac{x_u}{d} = \frac{415 \times 0.54}{54.27 \times 20} = 0.207$$

$$\text{Let } C_{fy} = 0.05 \quad \text{and } C_{fck} = 0.10$$

$$C_{xud} = \sqrt{(0.05^2 + 0.1^2)} = 0.112$$

$$C_{Mr} = \left[0.05^2 + \left(\frac{0.42 \times 0.207 \times 0.112}{1 - 0.42 \times 0.207} \right)^2 \right] = 0.052$$

$$\frac{M_r}{M_e} = \frac{229}{112.5} = 2.036$$

$$Z = \frac{1 - 2.036}{\sqrt{(0.1^2 + 2.036^2 \cdot 0.052^2)}} = -7.14$$

$$\text{Probability of failure} = \int_{-\infty}^{-7.14} e^{-x^2/2} dx = 0.32 \times 10^{-12}$$

5.4 Results and Discussion

Figure 5.1 is an interference diagram, used in deriving the probability of failure of a member. Figure 5.2 shows the typical interference diagram of a deteriorating member. The ordinate p_i shown in Fig. 5.2 is nothing but the interference area of that year. In this diagram mean value of M_e is taken as constant with time. However for actual case the diagram can be drawn if the magnitudes are specified. The interference area of any year is independent of the previous year's interference area, because the estimation of external moment and moment of resistance do not depend upon previous values. The application of the equations (5.13), (5.21) and (5.22) is discussed through examples. Four cases were studied. In the first case the probability of failure is constant in each year.

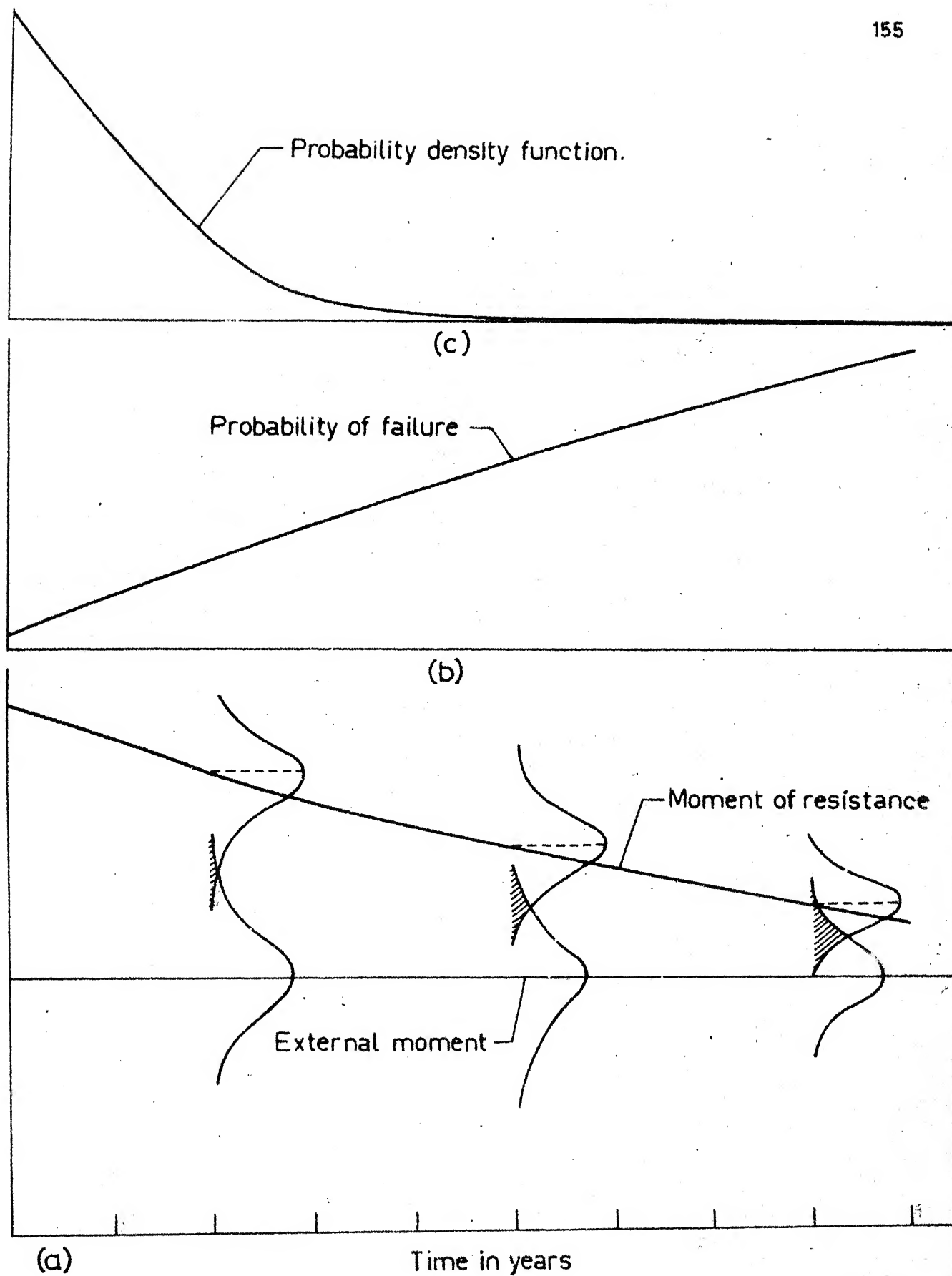


FIG. 5-2 TYPICAL (a) INTERFERENCE AREA, (b) PROBABILITY OF FAILURE AND (c) PROBABILITY DENSITY FUNCTION OF A

The second and third cases are that the probability of failure in each year remains constant upto 20 years, and it changes after 20 years due to reduction in strength by 20% in one case and due to increase in load by 20% in another case. The increased probability of failure remains constant in each year from 21st year onwards. In the fourth case external moment is constant throughout, and the moment of resistance decreases with time due to deterioration in concrete strength (Fig.5.3). In all the cases, the effect of survival for the first 10 years, 20 years, 30 years and 40 years on cumulative probability of failure was studied and tabulated in Tables 5.1 to 5.4.

In the following discussion, a comparison is made with the cumulative probability of failure calculated at the end of 50 years. From Table 5.1 one can see that a reduction in cumulative probability of failure is 20 %, 40 %, 60 % and 80 % for the cases that the beam has survived for the first 10 years, 20 years , 30 years and 40 years period respectively. If the probability of failure is constant (say $p_1 = p$) the cumulative probability of failure at the end of the t th year is approximately equal to $p \cdot t$. So is the case discussed above. Because, out of fifty years of life, if it has survived for the first 10 years, the cumulative probability of failure is $(50 - 10) p = 40 p$, a 20 % improvement. In cases two and

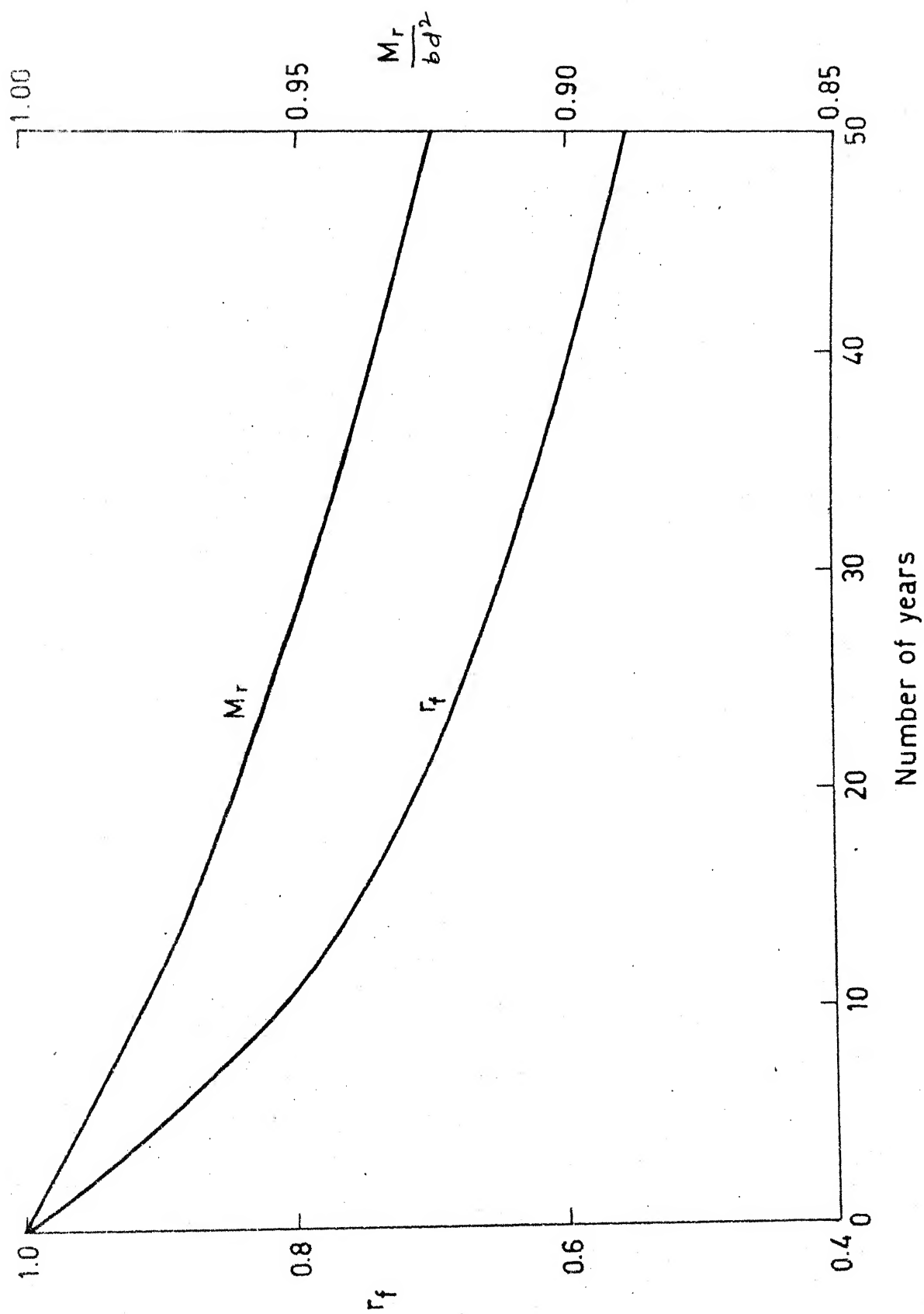


FIG. 5-3 DETERIORATION OF STRENGTH AND MOMENT CAPACITY

TABLE 5.1 : CUMULATIVE PROBABILITY OF FAILURE OF A BEAM
HAVING CONSTANT STRENGTH THROUGHOUT ITS LIFE

Sl. No.	Year	p_i	Cumulative probability of failure				
			Number of years survived				
			0	10	20	30	40
		$\times 10^{-9}$	$\times 10^{-9}$	$\times 10^{-9}$	$\times 10^{-9}$	$\times 10^{-9}$	$\times 10^{-9}$
1	1	0.354	0.354				
2	10	0.354	3.540				
3	11	0.354	3.892	0.354			
4	20	0.354	7.076	3.540			
5	21	0.354	7.429	3.892	0.354		
6	30	0.354	10.613	7.076	3.540		
7	31	0.354	10.967	7.429	3.892	0.354	
8	40	0.354	14.151	10.613	7.076	3.540	
9	41	0.354	14.505	10.967	7.429	3.892	0.354
10	50	0.354	17.689	14.151	10.613	7.076	3.540

TABLE 5.2 : CUMULATIVE PROBABILITY OF FAILURE OF A BEAM (STRENGTH DECREASES
BY 20 PERCENT AFTER 20 YEARS)

S.No.	Year	P_i	Cumulative Probability of failure				
			Number of years survived ($p_i = 0$)				
			0	10	20	30	40
			$x10^{-6}$	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$
1	1	0.000354	0.000354				
2	10	0.000354	0.003540				
3	11	0.000354	0.003890	0.000354			
4	20	0.000354	0.007080	0.003540			
5	21	42.339	42.347	42.343	42.339		
6	30	42.339	423.320	423.317	423.314		
7	31	42.339	465.640	465.640	465.630	42.339	
8	40	42.339	846.455	846.452	846.448	423.314	
9	41	42.339	888.759	888.755	888.752	465.640	42.339
10	50	42.339	1269.410	1269.400	1269.400	846.450	423.314

TABLE 5.3 : CUMULATIVE PROBABILITY OF FAILURE OF A BEAM (LOAD INCREASES BY 20 PERCENT AFTER 20 YEARS)

S.no.	Year	p_i	Cumulative Probability of Failure			
			Number of years survived ($p_i=0$)			
			0	10	20	30
			$x10^{-6}$	$x10^{-6}$	$x10^{-6}$	$x10^{-6}$
1	1	0.000354	0.000354			
2	10	0.000354	0.003540			
3	11	0.000354	0.003890	0.000354		
4	20	0.000354	0.007080	0.003540		
5	21	7.118	7.125	7.121	7.118	
6	30	7.118	71.180	71.177	71.173	
7	31	7.118	78.297	78.294	78.290	7.118
8	40	7.118	142.350	142.344	142.341	71.173
9	41	7.118	149.465	149.461	149.458	78.290
10	50	7.118	213.511	213.508	213.504	142.344
						71.173

TABLE 5.4 : CUMULATIVE PROBABILITY OF FAILURE OF AN EXPOSED BEAM

S.No.	Year	p_i	Cumulative Probability of failure				
			Number of years survived ($p_i=0$)				
			0	10	20	30	40
			$\times 10^{-12}$	$\times 10^{-12}$	$\times 10^{-12}$	$\times 10^{-12}$	$\times 10^{-12}$
1	1	0.320	0.320				
2	10	3.870	15.470				
3	11	4.930	20.400	4.933			
4	20	37.103	183.343	167.873			
5	21	45.733	229.125	213.656	45.783		
6	30	274.279	1512.000	1496.500	1328.620		
7	31	331.000	1843.000	1828.000	1660.00	331.0	
8	40	1632.000	9954.000	9938.000	9770.00	8442.0	
9	41	1928.000	11881.000	11866.000	11698.00	10369.0	1928.0
10	50	7318.000	53299.000	53283.000	53115.00	51787.0	43345.0

three the improvement is almost zero upto 20 years of survival and reduction of about 33.33 % and 67% for 30 years and 40 years of survival respectively, can be observed. In both cases the probability of failure is 3.54×10^{-10} upto 20 years and 42×10^{-6} in case two and 7×10^{-6} in case three. Since the probability of failure for the first 20 years is much lower than the later probability of failure, its effect is almost negligible in the improvement of cumulative probability of failure. In case of an exposed beam the first 20 years of survival shows practically no improvement. But there is an improvement of 2.84% and 18.7% for 30 and 40 years of survival respectively (Table 5.4, Fig. 5.4). For the exposed beam, reliability, cumulative probability of failure and hazard rate are plotted in Fig. 5.4. Sections a-a, b-b, c-c and d-d are the places where the known data is included. For example, at a-a, the curves are drawn after including the data on survival of the beam for the first 10 years. Similarly b-b, c-c and d-d are for 20 years, 30 years and 40 years of survival respectively.

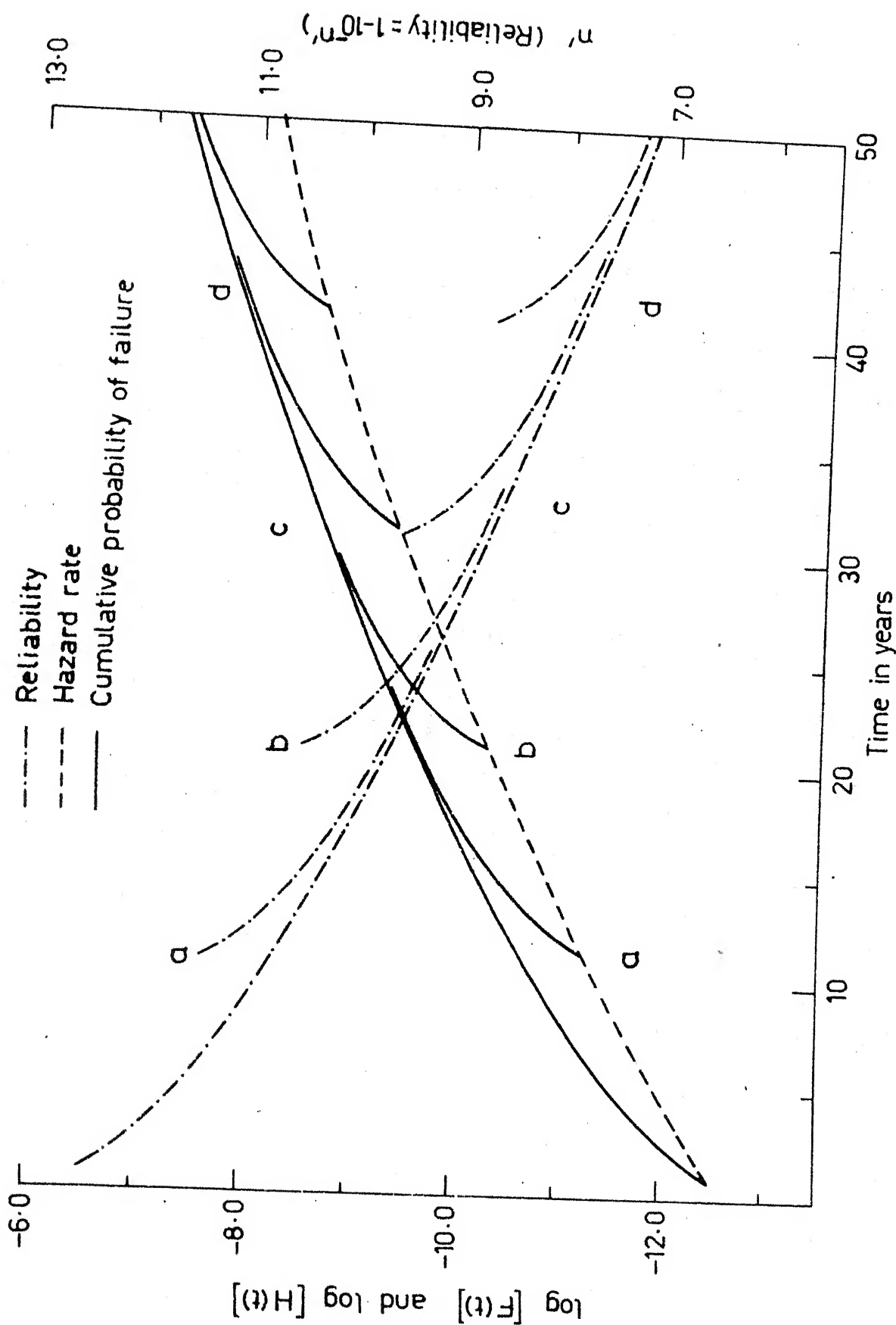


FIG. 5.4 CUMULATIVE PROBABILITY, RELIABILITY AND HAZARD RATE OF A DETERIORATING MEMBER (CASE IV)

CHAPTER VI

CONCLUSIONS

Acids react with cement paste and form calcium, aluminium and iron salts and silica gel. Sulphates react with cement paste and form insoluble and voluminous products. The volume increase causes the concrete to crack and spall. Due to sulphate attack, sulphotoaluminate corrosion and gypsum corrosion take place. In addition to these, magnesium corrosion also occurs when magnesium sulphate reacts with concrete. Sulphuric acid has dual affect. Concrete in 0.1% concentration of sulphuric acid increases in volume and shows a slight increase in strength. White fins are formed on the surface of the exposed concrete. In higher concentrations of the acid the dissolving attack dominates the sulphate attack. Therefore, the above observations can not be seen. However, yellow colour formation, during titration for determining strength of acid, is commonly observed. In 5% acid the mortar gets dissolved upto greater depths. Corrosion mechanisms can be used to develop deterioration indices. The stoichiometrical relations of various chemicals discussed in Chapter II are used to estimate the amount of a chemical required for saturated reaction with cement.

The experimental results on consumption of acid, weight loss, strength loss, reduction in size of a cube enabled to develop the expressions to estimate the strength and weight reduction factors; depth of penetration of acid, depth of deterioration of concrete and coefficient of variation of strength. All the above indices are expressed in terms of consumption of acid. Therefore, if once the acid concentration is specified, all the indices can be calculated. Not only that, the method adopted in estimating the above indices takes into account , variations in cement quality and quantity, water cement ratio and concentration of a chemical. The strength reduction factor is defined as the ratio of strength of an exposed concrete to 28 days cube strength of concrete. Similarly weight reduction factor is the ratio of weight of an exposed concrete to 28 days cube weight of concrete. Linear rate of reaction (with depth) is assumed which means a saturated reaction of concrete at the surface and no reaction at some depth (defined as depth of penetration of acid). If the acid penetrates to the level of reinforcement, corrosion will be initiated. So it is essential to design proper cover to protect the reinforced steel. The expression for depth of penetration of acid can be used in designing the cover. The strength reduction factor is useful in proportioning

the member for a specified exposure. Strength reduction factor and depth of penetration of acid can be used to estimate the life of a structure. In most of the exposed concretes, the damage starts with the surface and the life of a structure depends upon the extent of surface damage and depth of damaged concrete. Acid dissolves the surface mortar. Initially surface is scalled off. Further, mortar is dissolved deep inside. Due to this dissolving attack, concrete strength is not uniform and the standard deviation of strength increases with increase in deterioration. Because, the dissolving of mortar causes the concrete to get disintegrated and results in non-uniformity. Therefore the expression developed to estimate coefficient of variation of strength has considerable importance in probabilistic design of exposed beams.

In an under-reinforced concrete section the neutral axis depth is proportional to the percentage of tension reinforcement. But the moment capacity of the section increases at lower rate than the neutral axis depth. Therefore, the rate of decrease of moment capacity of an exposed beam increases with increase in tension reinforcement. It is found to be higher in over-reinforced sections than in under-reinforced sections. The ratio of moment capacity of an exposed beam to that of an unexposed one increases at

cost is 2.6 % , 5.6 % and 8.2% for 10, 100 and 1000 times increase in reliability respectively.

Probability of failure of a concrete beam (exposed to normal environment and fixed load pattern) is expected to be constant with time, if the variations in loading and strength remain constant and incorporated in the design through partial safety factors. Cumulative probability of failure at the age of 50 years of the structure is calculated and used for the purpose of comparison. The survival of a beam for the first 10 years is found to be ~~reducing~~ 20 percent. Similarly 20 years, 30 years and 40 years of survival has 40 % , 60% and 80% reduction respectively. Suppose the load on the beam is to be increased or the strength deteriorates suddenly, it is desirable to recalculate the cumulative probability of failure incorporating the actual conditions to assess whether the member is safe. It is found that 20% sudden decrease in moment capacity increases the probability of failure from $3(10^{-10})$ to $42(10^{-6})$. Similarly a 20 % sudden increase in load increases the probability of failure from $3(10^{-10})$ to $7(10^{-6})$. In exposed concrete beams, the strength of concrete deteriorates with time, which in turn reduces the moment capacity of a section. Therefore, the probability of failure increases year by year. Continuous deterioration of concrete, about 45 %,

increases the probability of failure from $3(10^{-11})$ to $8(10^{-7})$. Estimation of probability of failure incorporating the observed data is useful to decide for how long the structure can be put into use in future.

Scope for Further Study

Inspite of the research on properties of concrete being in progress for the last many decades, the real mechanism of behaviour of concrete in aggressive chemicals leaves wide scope for further research. The following areas can be explored.

- (1) Effect of sulphuric acid on different concretes made with various cements.
- (2) Extension of the procedure explained in this thesis for other acids, salts and seawater.
- (3) Effect of aggressive chemicals on reinforced steel embedded in concrete and its repercussions.
- (4) Measurement of actual depth of penetration of acid with sophisticated technique, to help in the design of members for durability considerations.

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APPENDIX - A

PROCEDURE FOR ESTIMATING V_R

Aggressive chemicals attack the hydrated cement compounds. For determining the quantity of a particular chemical required for a saturated reaction with one kN of cement, the data on the quantity of each hydrated compound available for reaction, and the stoichiometrical relations of hydrated compounds with a chemical are essential. Using the stoichiometrical relations in Eq. (2.15) to Eq. (2.21), the quantity of each hydrated compound can be calculated. After knowing the quantities of hydrated compounds, one can estimate the demand of sulphuric acid for a saturated reaction with one kN of cement, using Eq. (2.26) - Eq. (2.31). For the oxides of cement in Table 3.1, the $\text{CaSO}_4 \cdot 2\text{H}_2\text{O} / \text{C}_3\text{A}$ ratio is about 0.03, which favours the formation of monosulphoaluminate. Atomic weights of various compounds involved in the analysis are tabulated in Table A.1.

TABLE A. 1 : ATOMIC WEIGHTS OF COMPOUNDS USED IN
THE ANALYSIS

Sl.No.	Compound	Symbol	Atomic weight
1	H_2O	H	18
2	$3CaO.SiO_2$	C_3S	228
3	$2CaO.SiO_2$	C_2S	172
4	$Ca(OH)_2$	CH	74
5	$3CaO.Al_2O_3$	C_3A	270
6	$4CaO.Al_2O_3.Fe_2O_3$	C_4AF	486
7	$CaSO_4$	\bar{S}	136
8	$3CaO.Al_2O_3.3CaSO_4$	$C_3\bar{A}\bar{S}_3$	678
9	$CaO.Al_2O_3.CaSO_4$	$C_3\bar{A}\bar{S}$	406
10	$3CaO.2SiO_2.3H_2O$	CSH	342
11	$3CaO.Al_2O_3.6H_2O$	C_3AH_6	378
12	$3CaO.Al_2O_3.12H_2O$	C_3AH_{12}	486
13	$3CaO.Al_2O_3.CaSO_4.12H_2O$	$C_3\bar{A}\bar{S}H_{12}$	622
14	$3CaO.Al_2O_3.3CaSO_4.32H_2O$	$C_3\bar{A}\bar{S}_3H_{32}$	1254
15	$3CaO.Fe_2O_3.6H_2O$	C_3FH_6	436
16	H_2SO_4	$H\bar{S}$	98
17	$Al_2(SO_4)_3$	$\bar{A}\bar{S}_3$	342
18	$Fe_2(SO_4)_3$	$F\bar{S}_3$	400

Water required for complete hydration of 100 N of cement (compound wise) is as follows:

From Eq. (2.20), it can be seen that 2 molecules of C_3S reacts with 6 molecules of water.

Water required for complete hydration of C_3S

$$\begin{aligned}
 &= \frac{6 \text{ times the molecular weight of water}}{2 \text{ times the molecular weight of } C_3S} \% C_3S \\
 &= \frac{6 M_H}{2 M_{C_3S}} P_{C_3S} \quad (A.1)
 \end{aligned}$$

similarly water required for complete hydration of:

$$C_2S = \frac{4 M_H}{2 M_{C_2S}} P_{C_2S} \quad (\text{from Eq. (2.21)}) \quad (A.2)$$

$$C_3A = \frac{12 M_H}{M_{C_3A\bar{S}}} P_{C_3A} \quad (\text{from Eq. (2.16)}) \quad (A.3)$$

$$C_4AF = \frac{10 M_H}{M_{C_4AF} + 2M_{CH}} P_{C_4AF} \quad (\text{from Eq. (2.19)}) \quad (A.4)$$

$$\begin{aligned}
 \text{W/C ratio} &= \frac{M_H}{100} \left[\frac{3P_{C_3S}}{M_{C_3S}} + \frac{2P_{C_2S}}{M_{C_2S}} + \frac{12P_{C_3A}}{M_{C_3A\bar{S}}} + \frac{10P_{C_4AF}}{M_{C_4AF} + 2M_{CH}} \right] \\
 &\quad (A.5)
 \end{aligned}$$

where M refers to atomic weight of a compound, and

P for percentage . Subscript is as defined in Table A.1.

On hydration, two molecules of C_3S produce one molecule of calcium silicate hydrate (Eq. (2.20)).

Quantity of calcium silicate hydrate

$$Q_{CSH} = \frac{\text{Molecular weight of } \cancel{\text{lime}} \text{ CSH}}{2 \text{ times the molecular weight of } C_3S} \times \% C_3S$$

$$= \frac{M_{CSH}}{2M_{C_3S}} P_{C_3S} \quad (A.6)$$

Similarly from Eq. (2.21),

$$Q_{CSH} = M_{CSH} \left[\frac{3P_{C_3S}}{2M_{C_3S}} + \frac{P_{C_2S}}{2M_{C_2S}} \right] \quad (A.7)$$

From Eq. (2.19) to Eq. (2.21), quantity of lime

$$Q_{CH} = \frac{3M_{CH} P_{C_3S}}{2M_{C_3S}} + \frac{M_{CH} P_{C_2S}}{2M_{C_2S}} - \frac{2M_{CH} P_{C_4AF}}{M_{C_4AF}} \quad (A.8)$$

* Part of the lime will be consumed in the hydration of C_4AF .

From Eq. (2.19) Quantity of calcium aluminate hydrate

$$Q_{C_3AH_6} = \frac{M_{C_3AH_6} P_{C_4AF}}{M_{C_4AF}} \quad (A.9)$$

Quantity of calcium ferrite hydrate

$$Q_{C_3FH_6} = \frac{M_{C_3FH_6} P_{C_4AF}}{M_{C_4AF}} \quad (A.10)$$

From Eq. (2.16) , quantity of monosulphoaluminate

$$Q_{C3A\bar{S}H12} = \frac{M_{C3A\bar{S}H12} P_{C3A}}{M_{C3A\bar{S}}} \quad (A.11)$$

Three molecules of sulphuric acid reacts with one molecule of CSH (Eq. (2.26)).

∴ Sulphuric acid required for a saturated reaction with Q_{CSH}

$$\begin{aligned} A_{CSH} &= \frac{3 \text{ times the molecular weight of sulphuric acid}}{\text{Molecular weight of CSH}} Q_{CSH} \\ &= \frac{3 M_{HS}}{M_{CSH}} Q_{CSH} = 0.86 Q_{CSH} \quad (A.12) \end{aligned}$$

Similarly,

$$A_{CH} = \frac{M_{HS}}{M_{CH}} Q_{CH} = 1.32 Q_{CH} \text{ (from Eq. 2.27)} \quad (A.13)$$

$$\begin{aligned} A_{C3A\bar{S}H12} &= \frac{6 M_{HS}}{M_{C3A\bar{S}H12}} Q_{C3A\bar{S}H12} \\ &= 0.95 Q_{C3A\bar{S}H12} \text{ (from Eq. (2.30))} \quad (A.14) \end{aligned}$$

$$\begin{aligned} A_{C3AH6} &= \frac{6 M_{HS}}{M_{C3AH6}} Q_{C3AH6} = 1.56 Q_{C3AH6} \\ &\text{(from Eq. (2.29))} \quad (A.15) \end{aligned}$$

$$A_{C_3FH_6} = \frac{6M_{HS}}{M_{C_3FH_6}} Q_{C_3FH_6} = 1.35 Q_{C_3FH_6} \quad (\text{from Eq. (2.28)}) \quad (A.16)$$

Total sulphuric acid required /100N of cement

$$= A_{acid} = (0.86 Q_{CSH} + 1.32 Q_{CH} + 0.950 Q_{C_3ASH_{12}} + 1.56 Q_{C_3AH_6} + 1.35 Q_{C_3FH_6}) N / 100 N \text{ of cement} \quad (A.17)$$

Density of sulphuric acid is taken as 17.85 kN/m^3 . Therefore volume of acid required for saturated reaction with one Newton of cement (V_R) :

$$V_R = A_{acid} / (100 \times 17.85) = \frac{A_{acid}}{1785} \quad (A.18)$$

Similarly, one can calculate the demand of any chemical, if the stoichiometrical relations are known. For a cement given in Table 3.1, the compounds can be calculated using Eq. (2.2) - (2.9). If the values are substituted in Eq.(2.2) or Eq.(2.6) C_3^S comes out as a negative quantity, so it is taken as zero. The percentage of each compound is given in Table A.2.

TABLE A.2 : COMPOUND COMPOSITION OF A CEMENT GIVEN IN
TABLE 3.1.

Compound	Weight in Percentage
C_3^S	0.00
C_2^S	78.34
C_3^A	10.75
C_4^{AF}	13.08

Water cement ratio and demand of sulphuric acid for a saturated reaction with one Newton of cement, for the compounds given in Table A.2 are as follows:

$$\begin{aligned}
 W/C &= \frac{M_H}{100} \left(\frac{3P_{C3S}}{M_{C3S}} + \frac{2P_{C2S}}{M_{C2S}} + \frac{12P_{C3A}}{M_{C3AS}} + \frac{10P_{C4AF}}{M_{C4AF} + 2M_{CH}} \right) \\
 &= \frac{18}{100} \left(\frac{0}{228} + \frac{2 \times 78.34}{172} + \frac{12 \times 10.75}{406} + \frac{10 \times 13.08}{486 + 2 \times 74} \right) = 0.29
 \end{aligned}$$

$$\begin{aligned}
 Q_{CSH} &= M_{CSH} \left(\frac{P_{C3S}}{2M_{C3S}} + \frac{P_{C2S}}{2M_{C2S}} \right) \\
 &= 342 \left(0 + \frac{78.34}{2 \times 172} \right) = 77.88 \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 Q_{CH} &= M_{CH} \left(\frac{3P_{C3S}}{2M_{C3S}} + \frac{P_{C2S}}{2M_{C2S}} - \frac{2P_{C4AF}}{M_{C4AF}} \right) \\
 &= 74 \left(0 + \frac{78.34}{2 \times 172} - \frac{2 \times 13.08}{486} \right) = 12.37 \text{ N}
 \end{aligned}$$

$$Q_{C3ASH12} = \frac{M_{C3ASH12} P_{C3A}}{M_{C3AS}} = \frac{622 \times 10.75}{406} = 16.47 \text{ N}$$

$$Q_{C3AH6} = \frac{M_{C2AH6} P_{C4AF}}{M_{C4AF}} = \frac{378 \times 13.08}{486} = 10.17 \text{ N}$$

$$Q_{C3FH6} = \frac{M_{C3FH6} P_{C4AF}}{M_{C4AF}} = \frac{436 \times 13.08}{486} = 11.73 \text{ N}$$

$$\therefore V_R = \frac{1}{1785} (0.86 Q_{CSH} + 1.32 Q_{CH} + 0.95 Q_{C3ASH12} \\ + 1.56 Q_{C3AH6} + 1.35 Q_{C3FH6})$$

$$= \frac{(0.86 \times 77.88 + 1.32 \times 12.87 + 0.95 \times 16.47 \\ + 1.56 \times 10.17 + 1.35 \times 11.73)}{1785}$$

$$= 0.0734 \text{ m}^3/\text{kN} \text{ or } 73.4 \text{ l/kN.}$$

APPENDIX - B

ULTIMATE MOMENT CAPACITY OF A SECTION
(WITHOUT PARTIAL SAFETY FACTORS)

Considering Fig. B.1 :

$$\begin{aligned} \text{Area of OABCE} = C &= 0.67 f_{ck} \left[x_r + \frac{2}{3} (x_u - x_r) \right] \\ &= \frac{34}{63} f_{ck} x_u \end{aligned} \quad (\text{B.1})$$

$$\begin{aligned} \text{Moment of the area OABCE about BC} &= C x_c \\ &= 0.67 f_{ck} \cdot \frac{3}{7} x_u \cdot \frac{3}{14} x_u \\ &\quad + \frac{4}{9} f_{ck} \cdot \frac{4}{7} x_u \left(\frac{3}{8} \cdot \frac{4}{7} x_u + \frac{3}{7} x_u \right) \\ &= \frac{11}{49} f_{ck} x_u^2 \end{aligned} \quad (\text{B.2})$$

Dividing Eq. (B.2) by Eq. (B.1), gives

$$x_c = \frac{0.225 f_{ck} x_u^2}{0.543 f_{ck} x_u} = 0.42 x_u \quad (\text{B.3})$$

$$\text{Total compressive force} = C_u = 0.543 f_{ck} x_u b \quad (\text{B.4})$$

$$\text{Total tensile force} \equiv T = f_y A_{st} \quad (\text{B.5})$$

$$\text{Limiting neutral axis depth} = k_b = \frac{\epsilon_{cu}}{\epsilon_{cu} + \epsilon_{sb}} \quad (\text{B.6})$$

$$\text{where } \epsilon_{sb} = \frac{f_y}{E_s} + 0.002$$

E_s = Young's modulus of steel, 200 Gpa, and

$$\epsilon_{cu} = 0.0035$$

Substituting the above values in Eq.(B.6) :

$$\begin{aligned} k_b &= \frac{0.0035}{0.0035 + \frac{f_y}{2 \times 10^5} + 0.002} \\ &= \frac{700}{1100 + f_y} \end{aligned} \quad (B.7)$$

If $\frac{x_u}{d} > k_b$,

$$\begin{aligned} \text{Moment of resistance} &= M_r = C_u (d - x_c) \\ &= 0.5427 f_{ck} b d^2 k_b (1 - 0.42 k_b) \end{aligned} \quad (B.3)$$

$$\begin{aligned} \text{Moment of resistance} &= T (d - x_c) \\ &= f_y A_{st} d \left(1 - 0.42 \frac{x_u}{d} \right) \end{aligned} \quad (B.9)$$

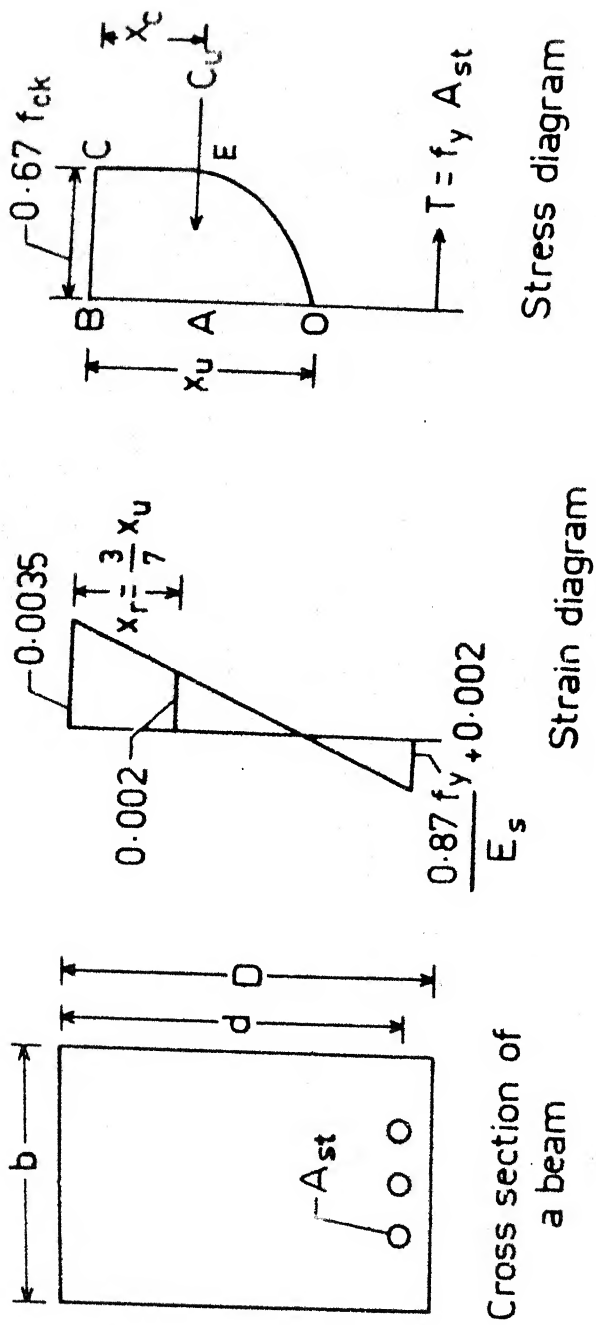


FIG. B.1 SINGLY REINFORCED RECTANGULAR SECTION

$f_{ck} = 20 \text{ N/mm}^2$

